Hydrodynamics for the Beam Energy Scan

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What we want to achieve in the BES

- Rigorously extract the equation of state of nuclear matter and determine existence and position of the critical point
- Quantify the characteristics of the Chiral Magnetic Effect
- This is done by:
    - Developing a comprehensive framework
        - Perform fundamental calculations (lattice QCD, etc.)
        - Determine which observables are most sensitive to critical behavior
        - Study dependence on parameters
        - Understand backgrounds
    - Comparing results to experimental data from BESII

Importance of Hydrodynamics for BEST

- Hydrodynamics will provide underlying framework for all calculations that are to be connected to experimental data:
    - Effects of the equation of state enter through the evolution of the system and its effect on final particle spectra
    - Non-equilibrium evolution of cumulants of critical fluctuations on the hydrodynamic bulk evolution background

      Swagato Mukherjee, Raju Venugopalan, Yi Yin, Phys.Rev. C92 (2015) no.3, 034912
    - Dynamic calculations of the CME are done in an extended hydrodynamic framework - (e.g. magnetohydrodynamics)
- Fluctuating initial states and off-equilibrium early-time evolution are crucial ingredients for a complete hydrodynamic framework

Towards hydrodynamics for the BES

Make hydrodynamic code MUSIC ready for the BES by

- including finite net-baryon density
  - equation of state (EoS) at finite net-baryon density
  - baryon diffusion
  - test EoS with critical point

- coupling it to a **dynamical** fluctuating initial state appropriate for long medium formation times

- including a proper treatment of strangeness and isospin (not yet in this talk)

Equation of state

Construct EoS at finite μ_B using Taylor expanded lattice data:

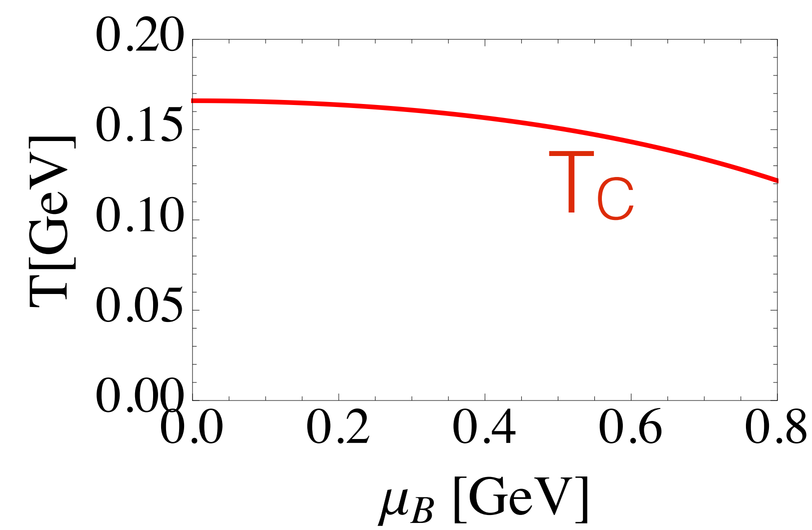
Hadron resonance gas

Lattice QCD

interpolate

connect smoothly at T_C

(**c**onnecting temperature)



$$\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2}\chi_B^{(2)}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{4!}\chi_B^{(4)}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^6\right]$$

Currently using data for parameters P_0^{lat} and $\chi_B^{(2)}$ from:

Borsanyi et al, JHEP1011, 077 (2010); JHEP1201, 138 (2012)

$\chi_B^{(4)}$ from the ratio $\chi_B^{(4)}/\chi_B^{(2)}$ in a HRG and Parton gas model

Equation of state

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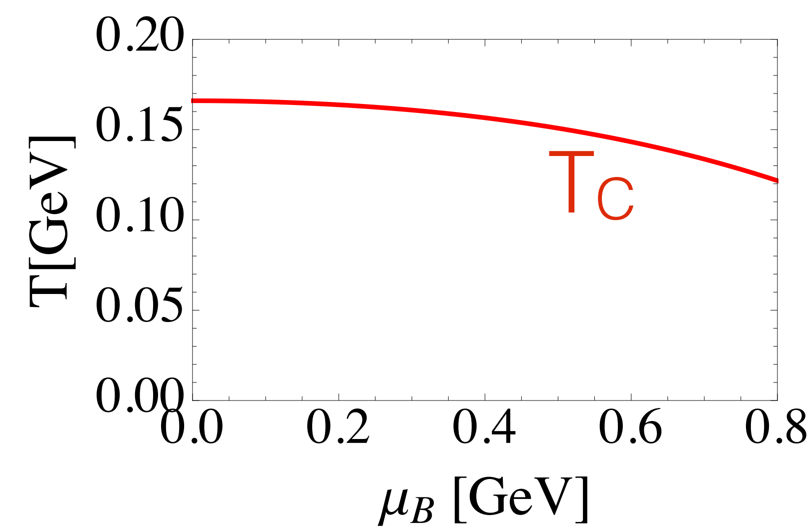


$$\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2}\chi_B^{(2)}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{4!}\chi_B^{(4)}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^6\right]$$

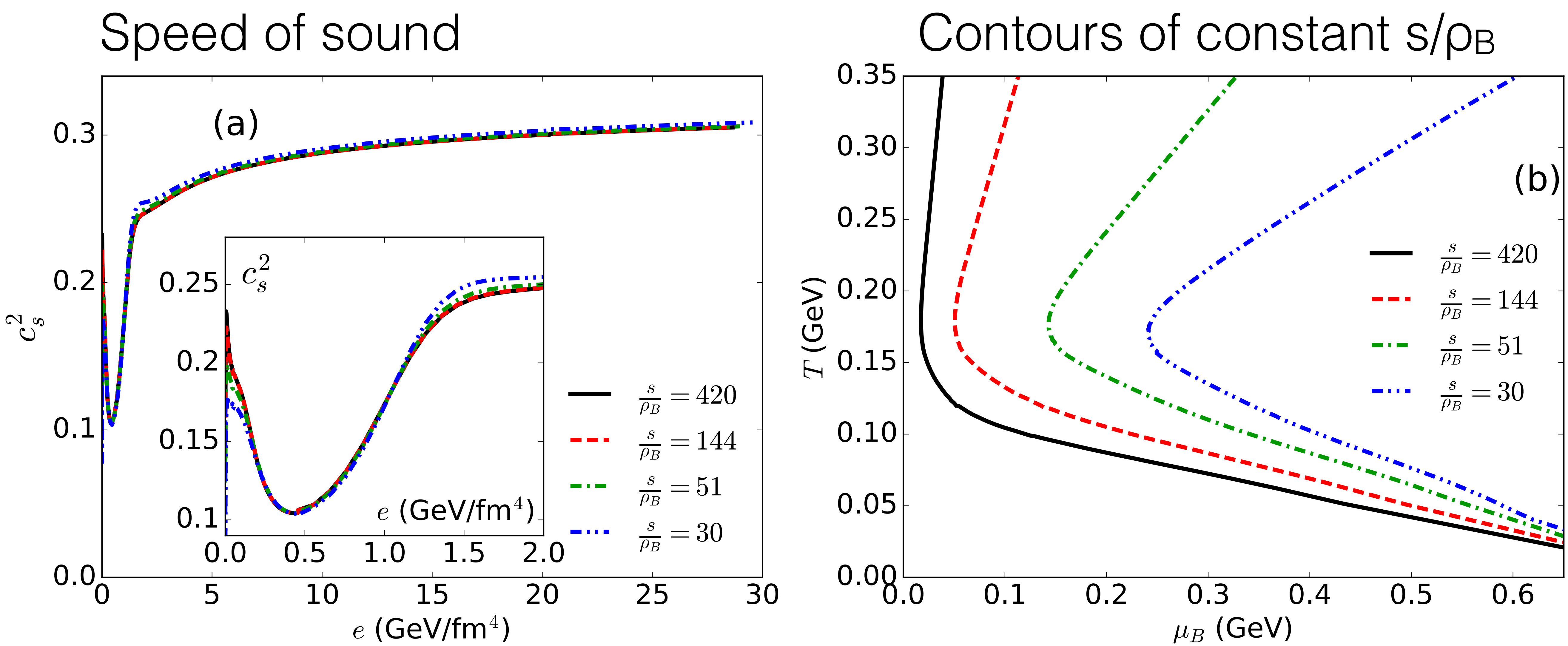
Working on a version including lattice data from

A. Bazavov et al. [HotQCD Collaboration], Phys. Rev. D 90, 094503 (2014)

A. Bazavov et al. [HotQCD Collaboration], Phys. Rev. D 86, 034509 (2012)

H.-T. Ding, S. Mukherjee, H. Ohno, P. Petreczky and H.-P. Schadler, Phys. Rev. D 92, 074043 (2015)

Equation of state

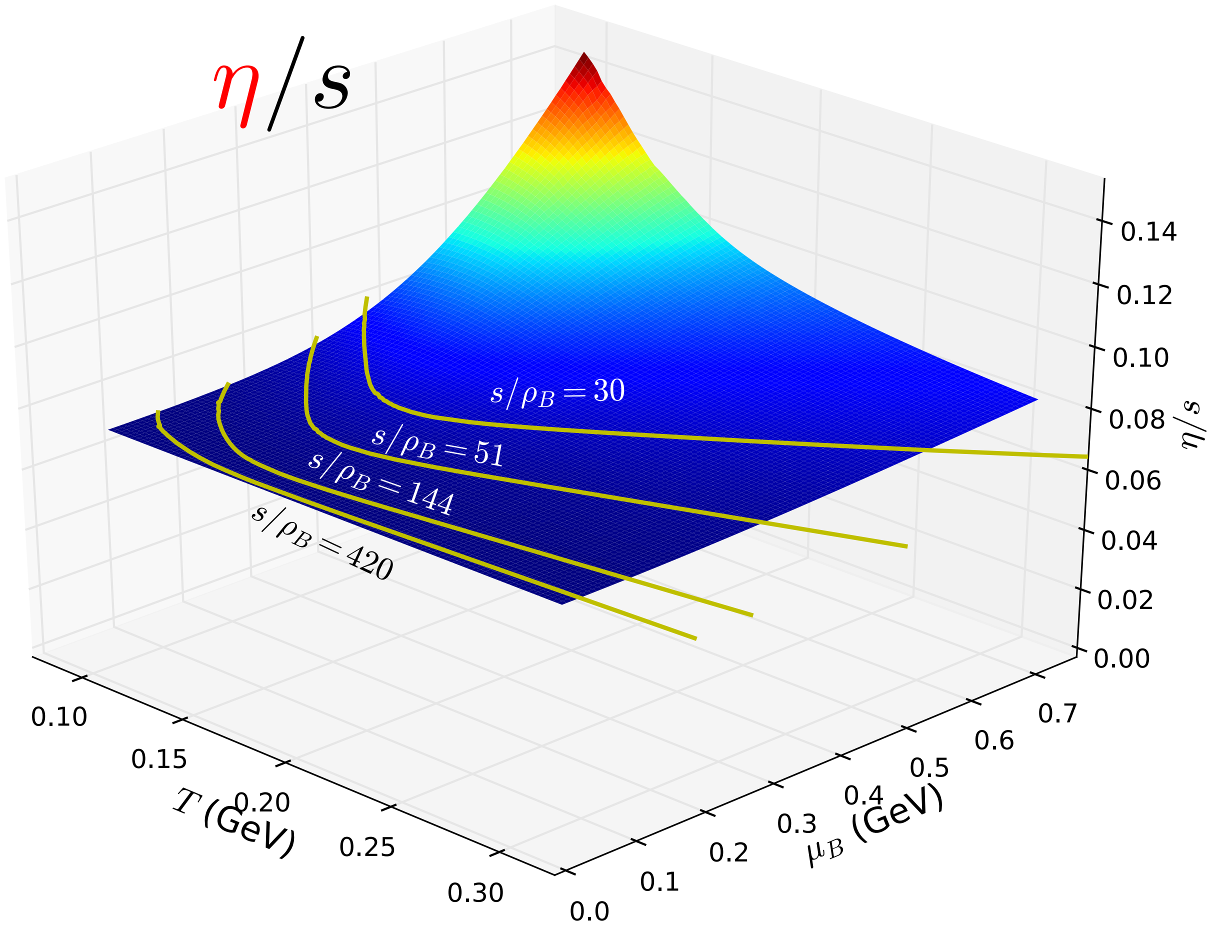


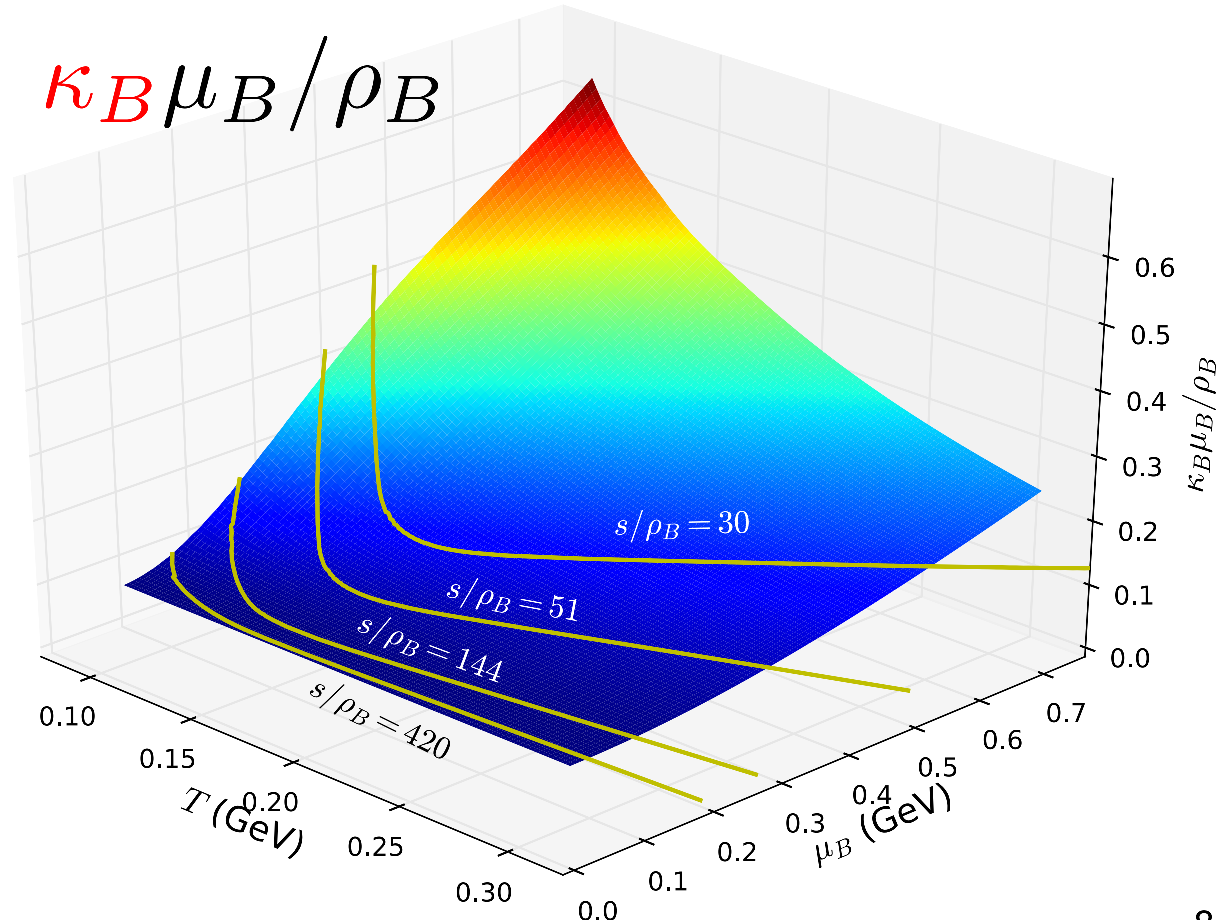
Transport coefficients including baryon diffusion

$$\Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}) - \frac{\delta_{\pi\pi}}{\tau_\pi}\pi^{\mu\nu}\theta - \frac{\tau_{\pi\pi}}{\tau_\pi}\pi^{\lambda\langle\mu}\sigma^{\nu\rangle}{}_\lambda + \frac{\phi_7}{\tau_\pi}\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}$$

$$\Delta^{\mu\nu} Dq_\nu = -\frac{1}{\tau_q}(q^\mu - \kappa\nabla^\mu\frac{\mu_B}{T}) - \frac{\delta_{qq}}{\tau_q}q^\mu\theta - \frac{\lambda_{qq}}{\tau_q}q_\nu\sigma^{\mu\nu}$$

Choose $\tau_\pi = \tau_q = \dfrac{0.4}{T}$, $\dfrac{\eta T}{e+\mathcal{P}} = 0.08$, $\kappa_B = \dfrac{C_B}{T}n_B\left(\dfrac{1}{3}\coth(\alpha_B) - \dfrac{n_B T}{e+\mathcal{P}}\right)$

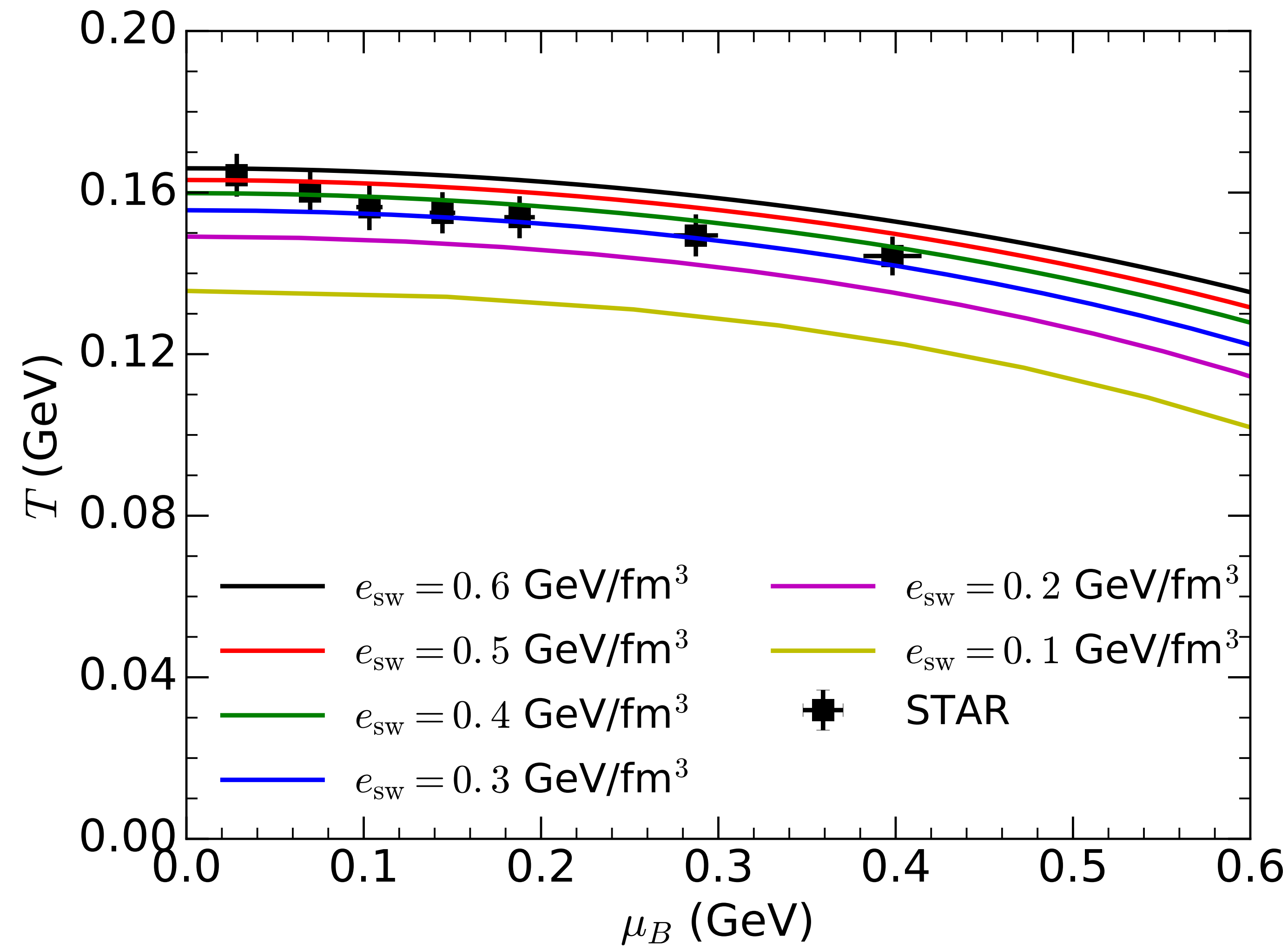




Conversion to particles

Switching-surface
is determined
at constant energy density

Then follows UrQMD



Cooper-Frye conversion:

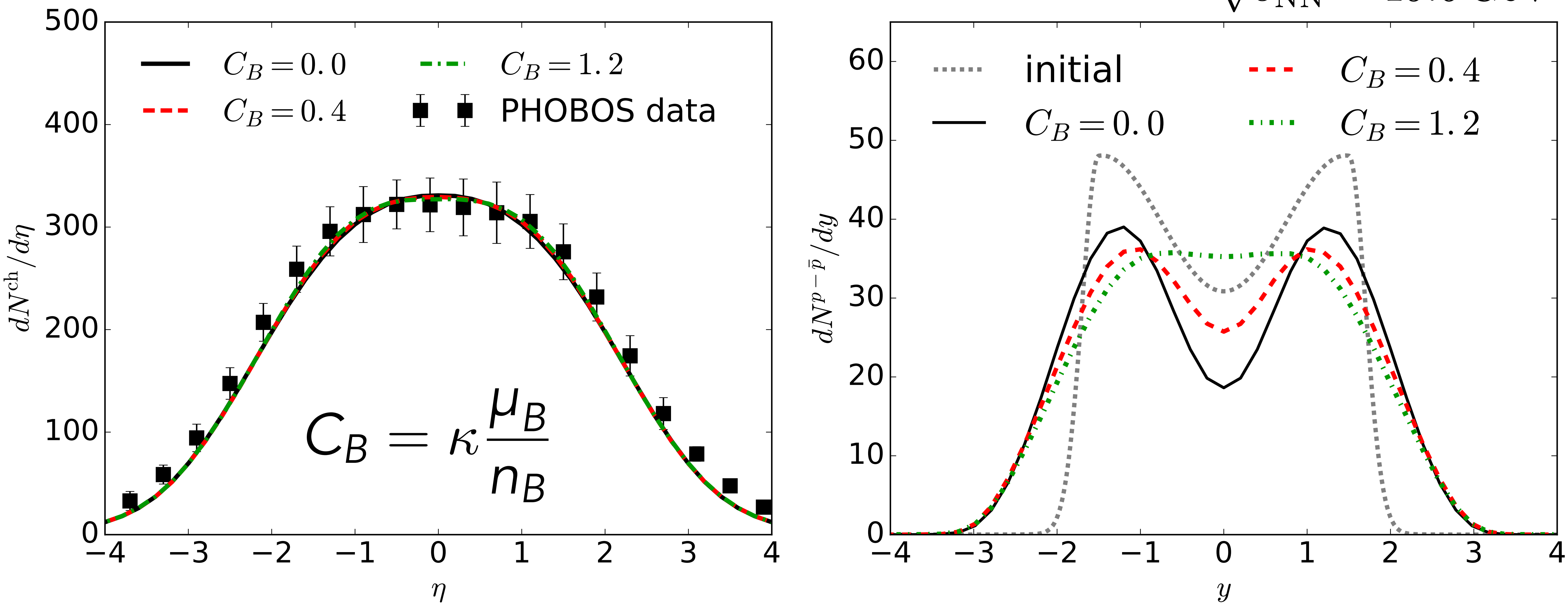
$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int p^\mu d^3\sigma_\mu(x)\left(f_0(x,p) + \delta f(x,p)\right)$$

$$f_0^i(x,p) = \frac{1}{e^{(E-b_i\mu_B(x))/T(x)} \pm 1}$$

Effects of net baryon diffusion on particle yields

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, arXiv:1704.04109

$\sqrt{s_{\mathrm{NN}}} = 19.6\,\mathrm{GeV}$

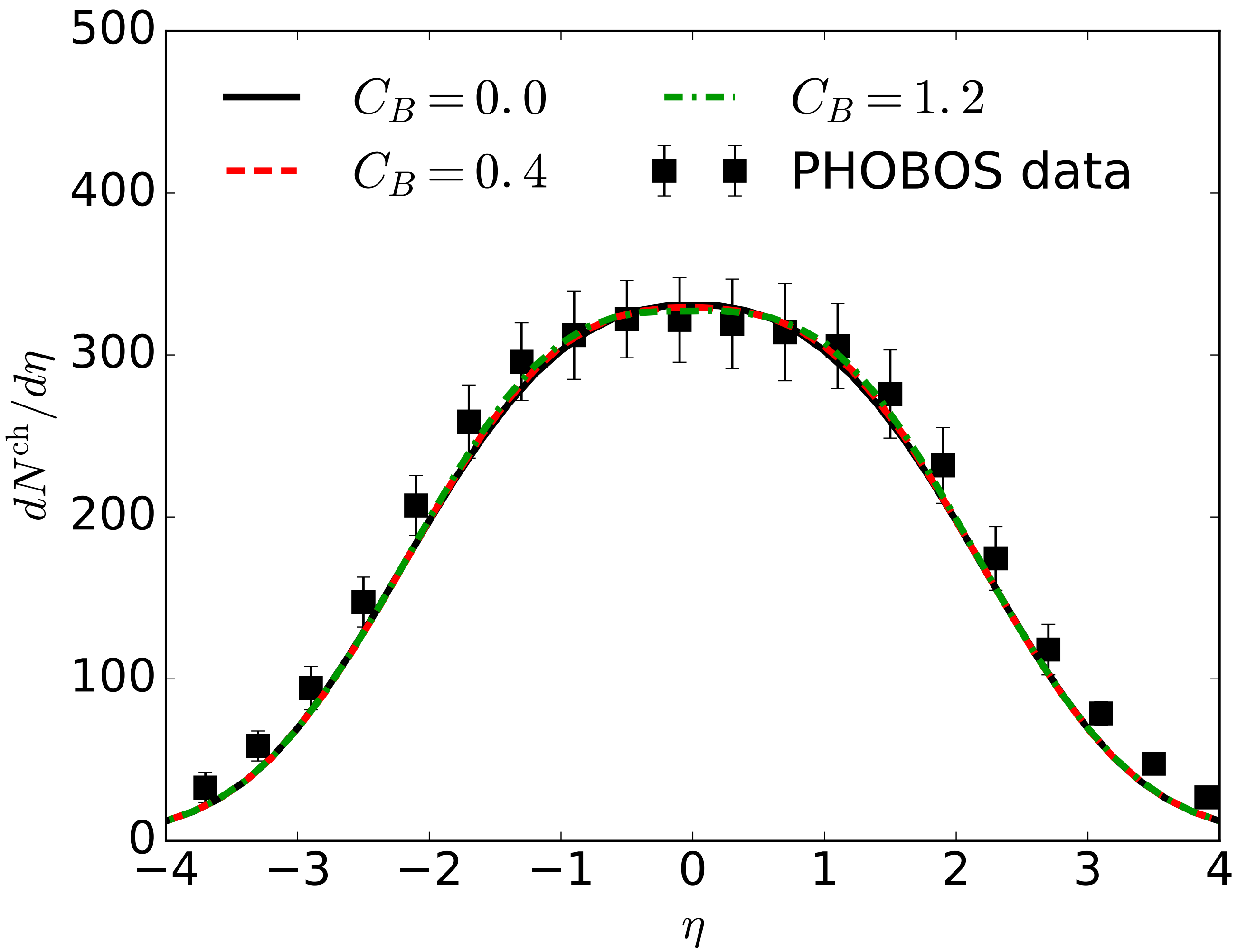


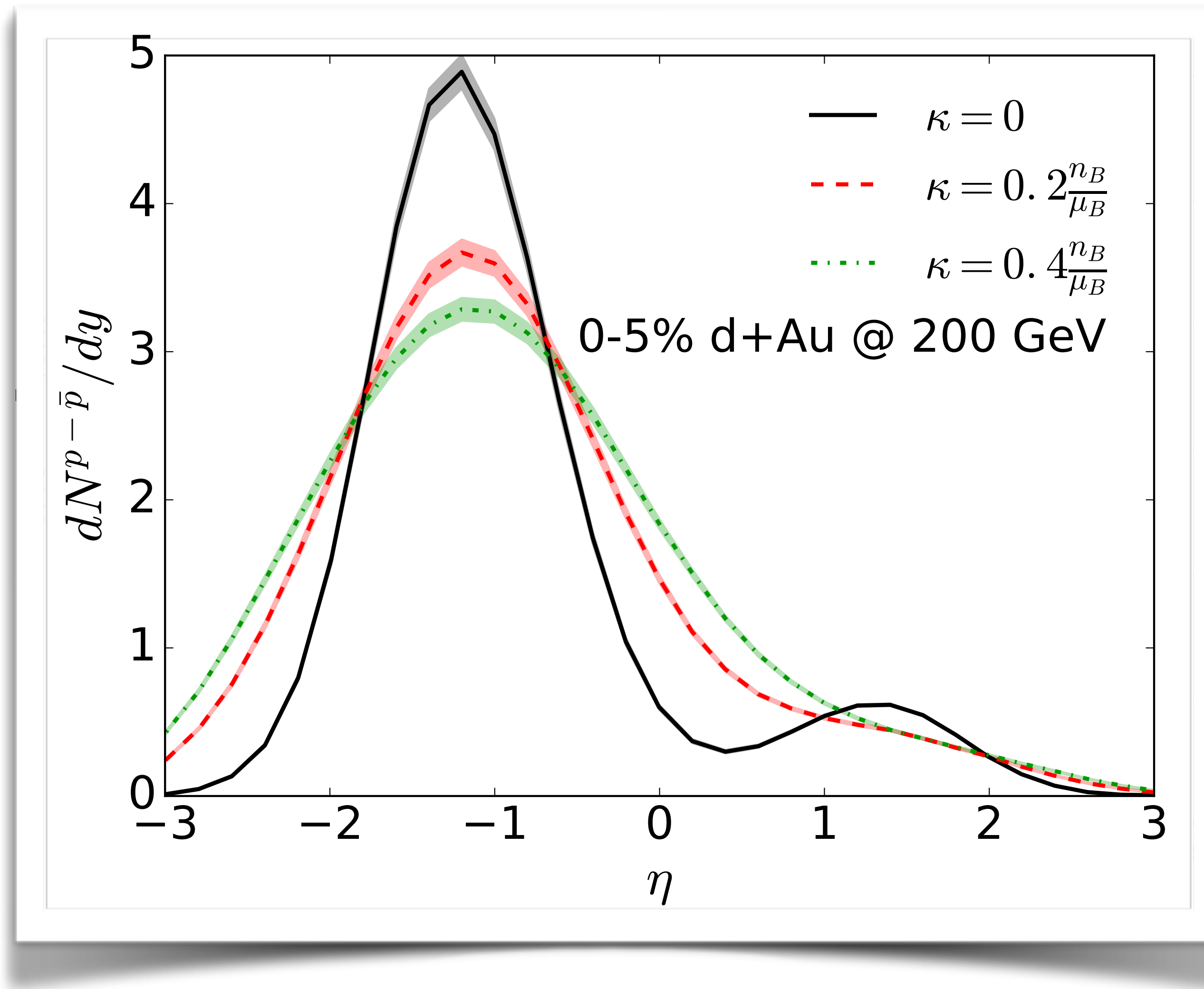
- Net baryon diffusion has little effect on charged hadron pseudo-rapidity distribution
- More net baryon numbers are transported to mid-rapidity with a larger diffusion constant

Constraints on net baryon diffusion and initial condition

Effects of net baryon diffusion on particle yields

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, arXiv:1704.04109





- Net baryon diffusion has little effect on charged hadron pseudo-rapidity distribution
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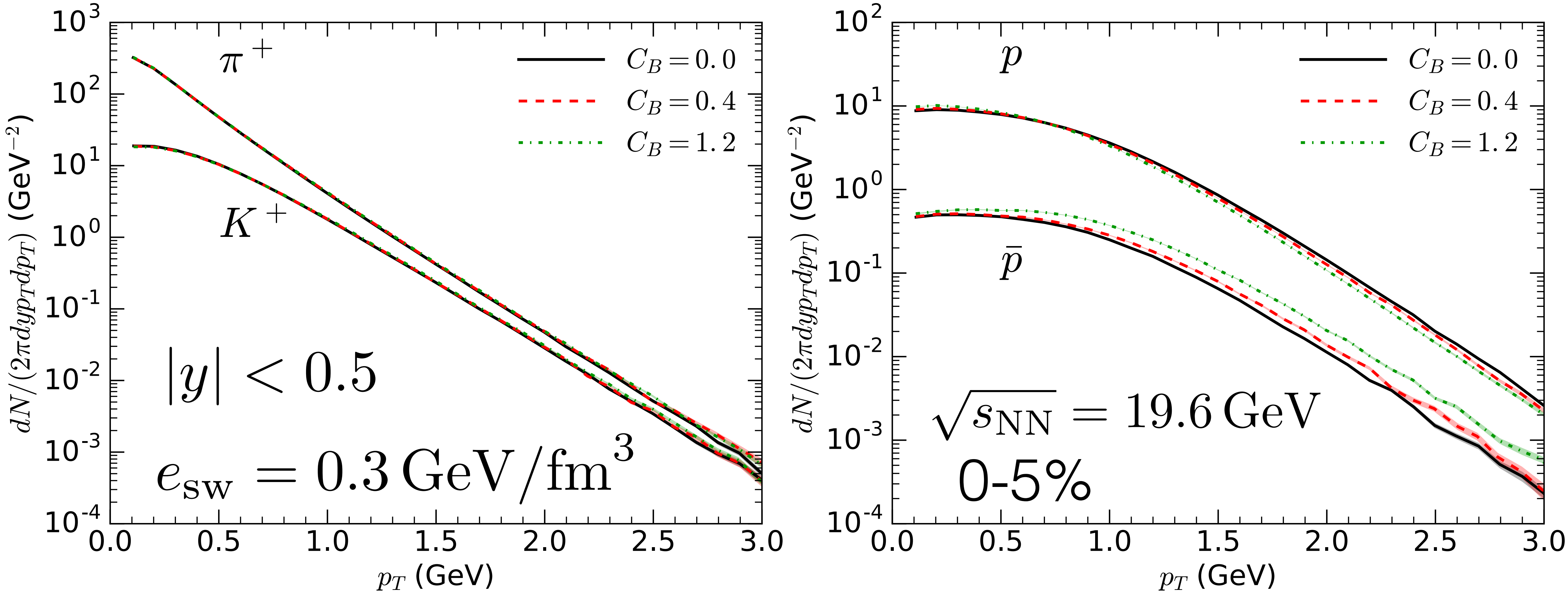
Constraints on net baryon diffusion and initial condition

Effect on transverse momentum spectra

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation

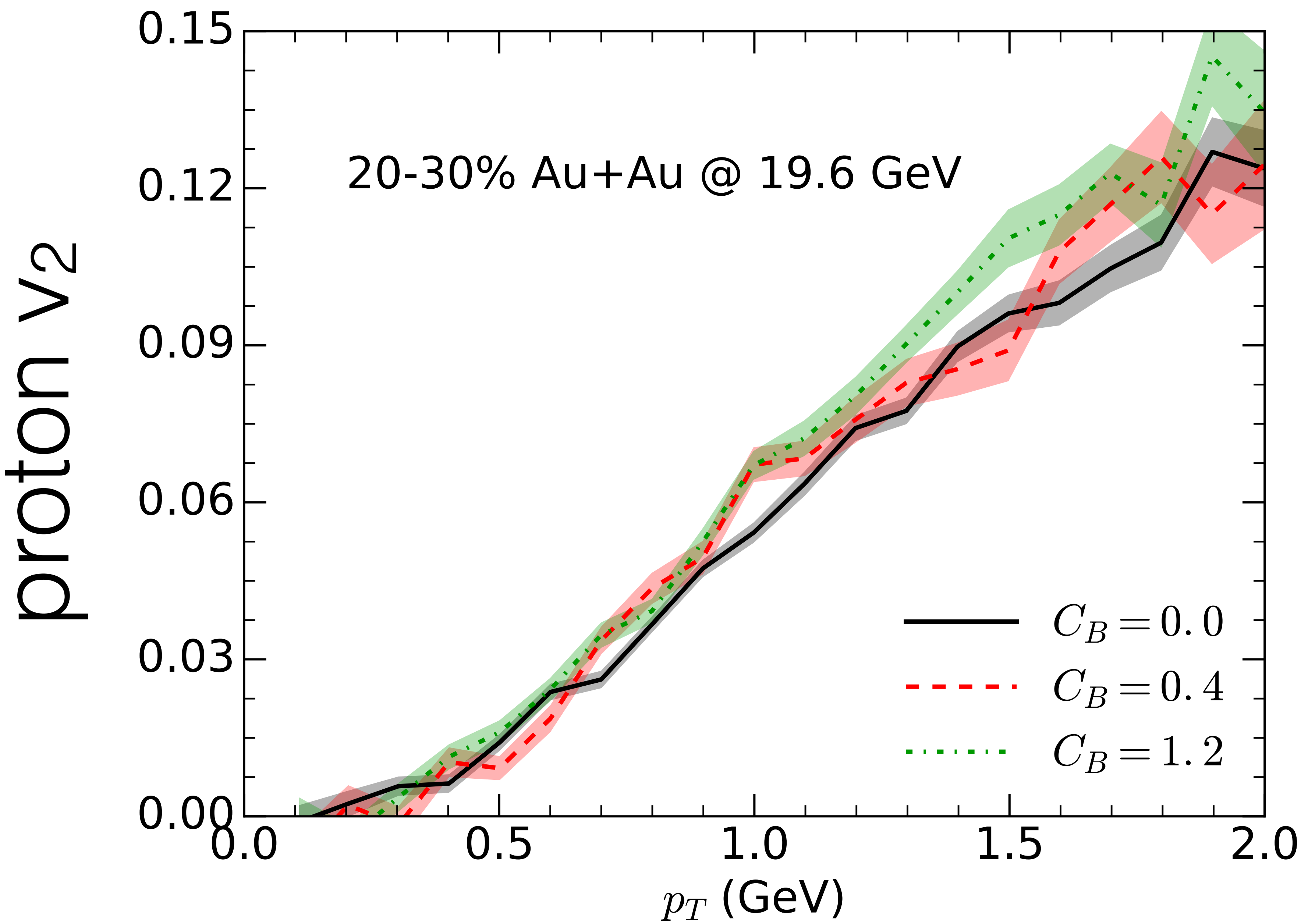


- Net baryon diffusion results in flatter spectra for anti-proton compared to proton's

	$C_B = 0.0$	$C_B = 0.4$	$C_B = 1.2$
$\langle p_\perp \rangle^{\bar{p}} - \langle p_\perp \rangle^{p}$ (GeV)	0.046	0.091	0.158

Effects of net baryon diffusion on elliptic flow

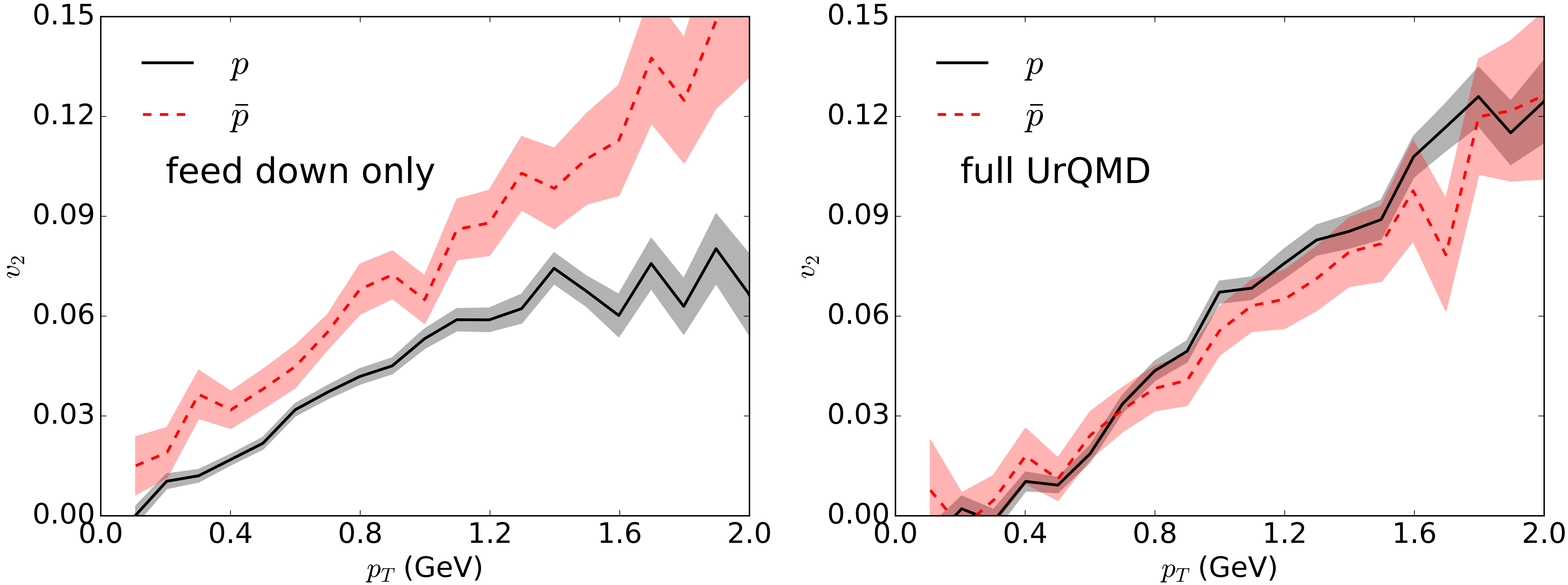
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- Hadronic scatterings wash out most of the diffusion effects on identified particle v_2

$v_2(p)$ vs. $v_2(\bar{p})$

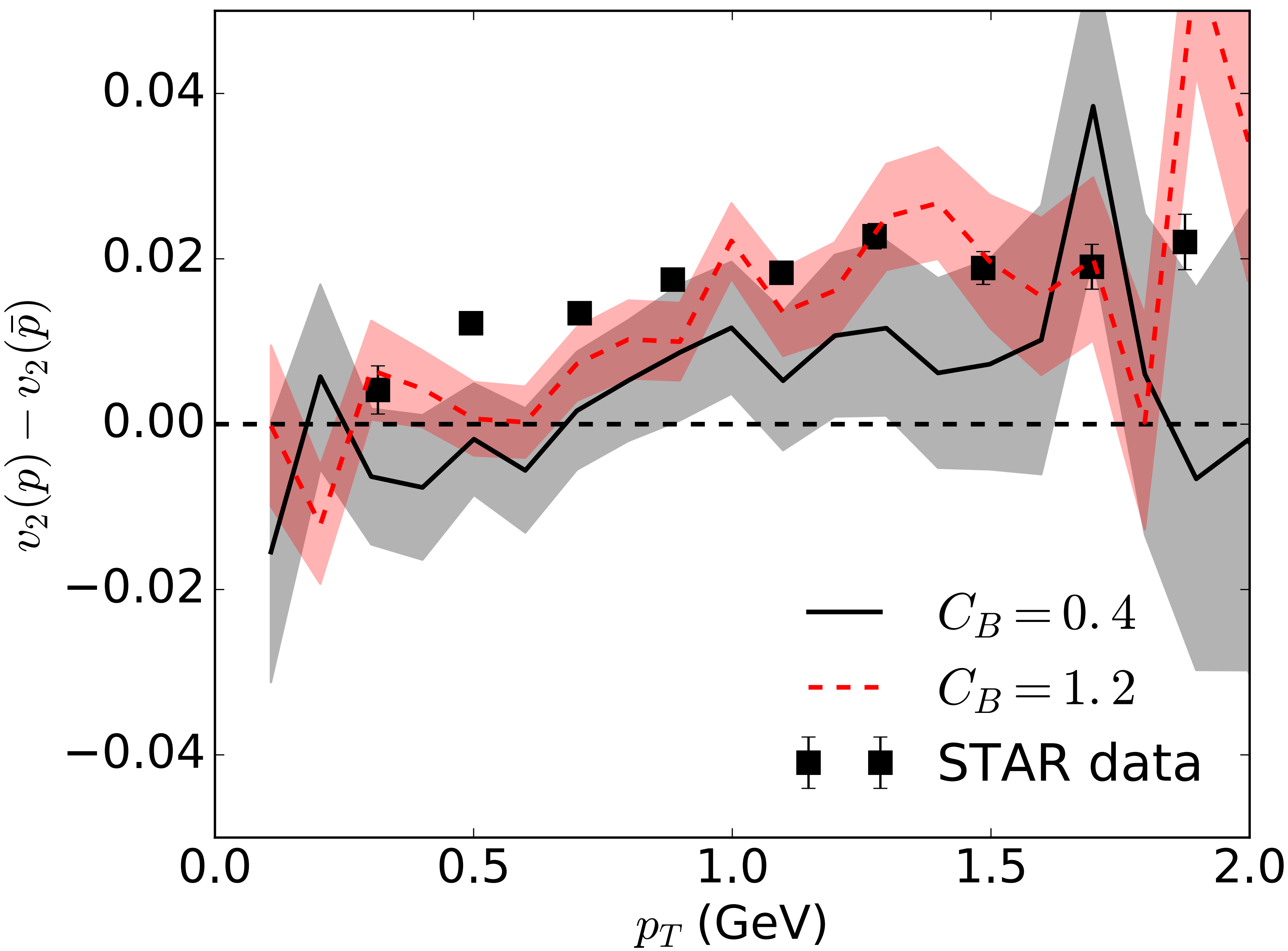
C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- Late stage hadronic scatterings correct the $v_2(p_T)$ ordering in our hybrid simulations

$v_2(p)$ vs. $v_2(\bar{p})$

C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, in preparation



- Larger net baryon diffusion constant leads to a larger $v_2(p) - v_2(\bar{p})$

3D initial conditions

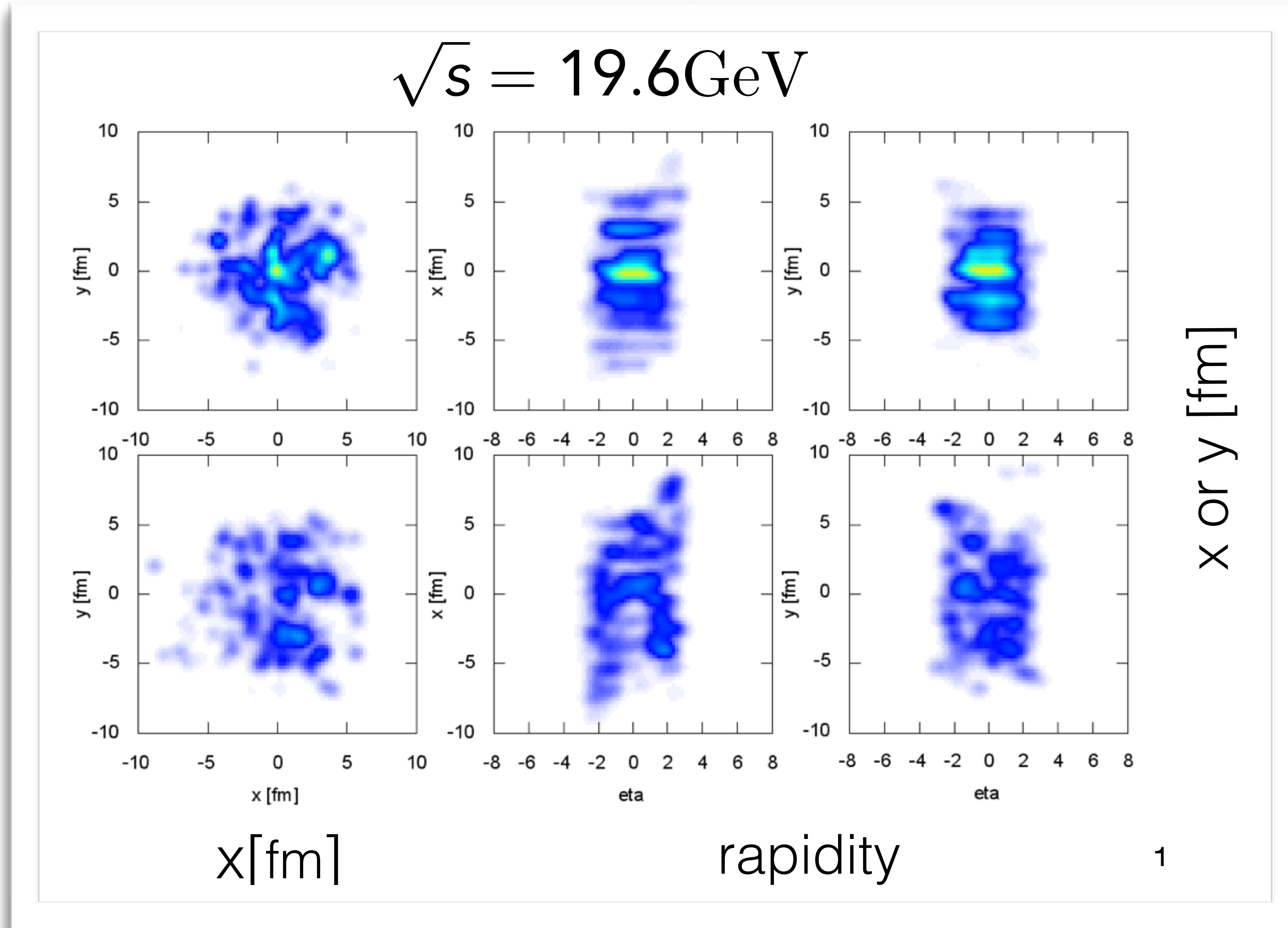
3D initial conditions generated by

- UrQMD, AMPT, EPOS or similar model
- Simple MC-Glauber type model on nucleon or quark level with e.g.
  - Lexus model for the longitudinal fluctuations (Monnai, Schenke)
  - Strings of random longitudinal length (Broniowski, Bozek, …)
- JIMWLK + IP-Glasma (upside: closest to QCD, downside: no net baryon density without extension; use of high energy limit)
- Strong coupling (van der Schee…)

We should employ different initial state models (with different parameter sets) and study the sensitivity of observables

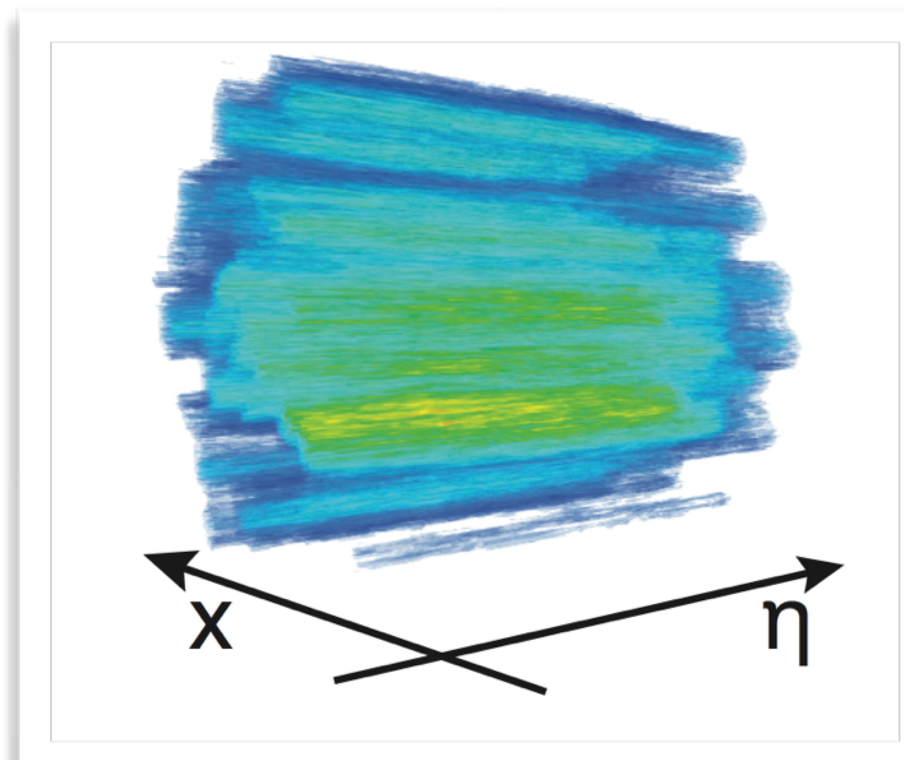
What degree of dynamics will be needed before we start the hydro?

3D initial conditions



Quark 3DMC-Glauber

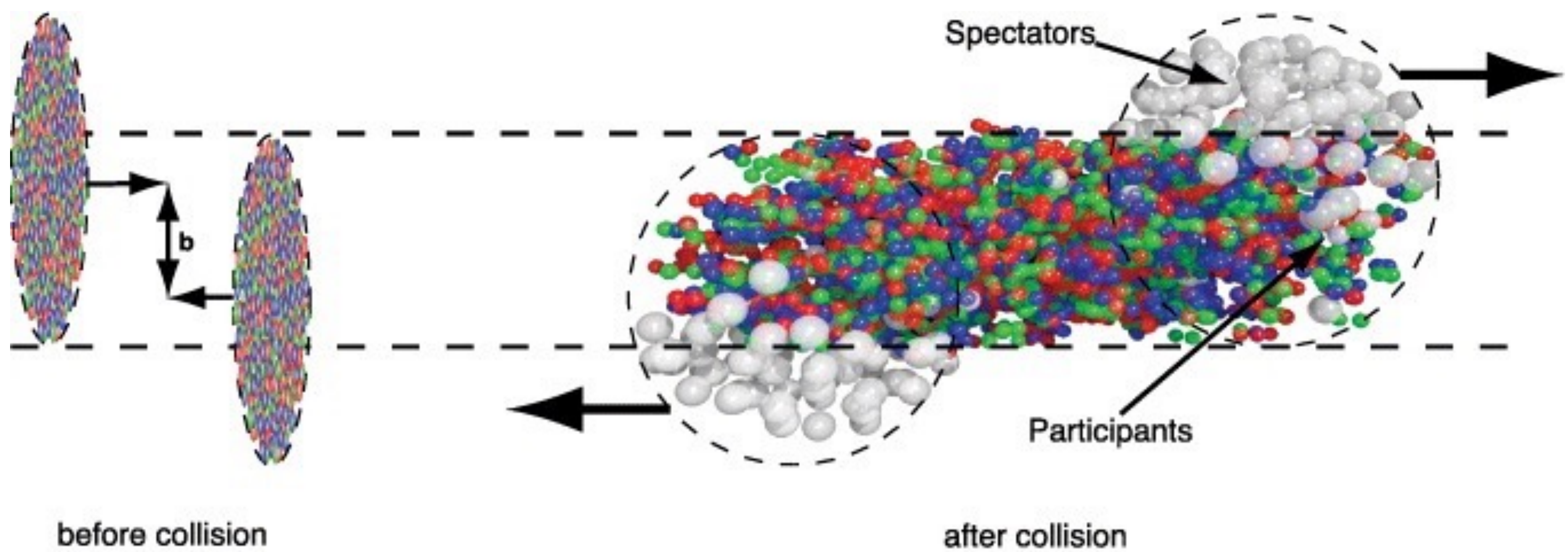
A. Monnai, B. Schenke, Phys.Lett. B752 (2016) 317-321



3D-Glasma

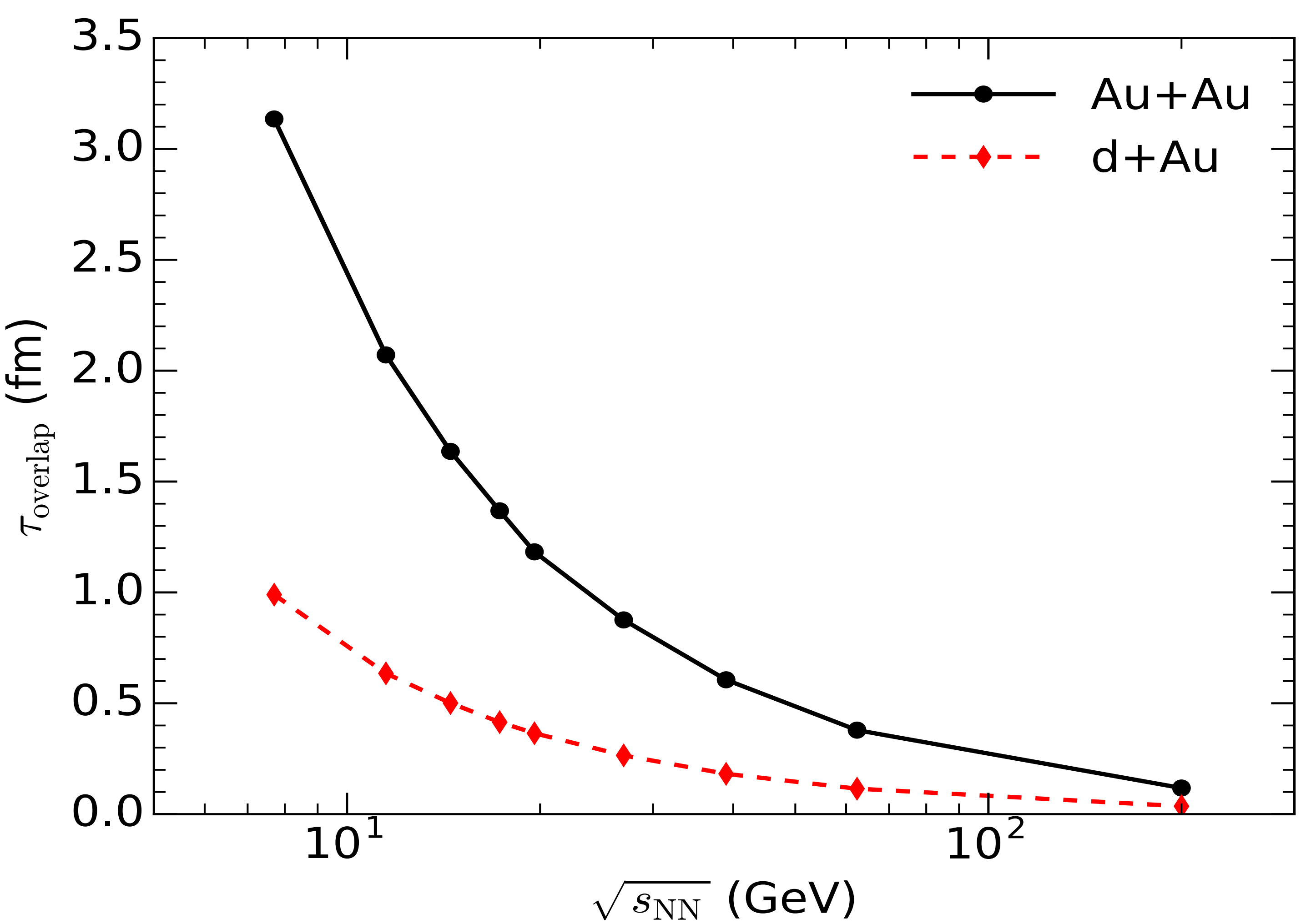
B. Schenke, S. Schlichting
Phys.Rev. C94 (2016) no.4, 044907

Need for dynamic initial states



Two nuclei overlapping time

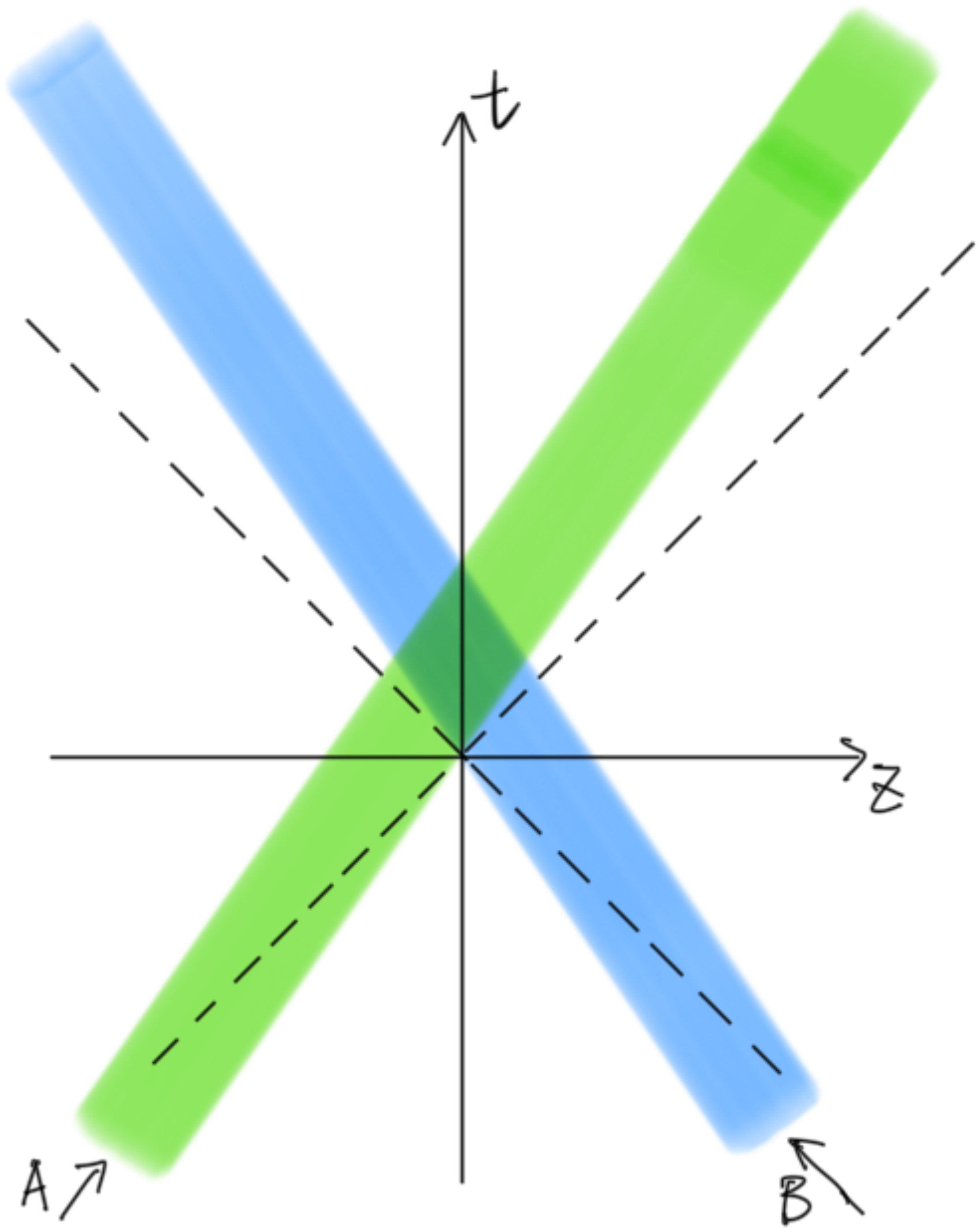
$$\tau \sim \frac{2R}{\gamma v_z}$$



- Nuclei overlapping time is large at low collision energy

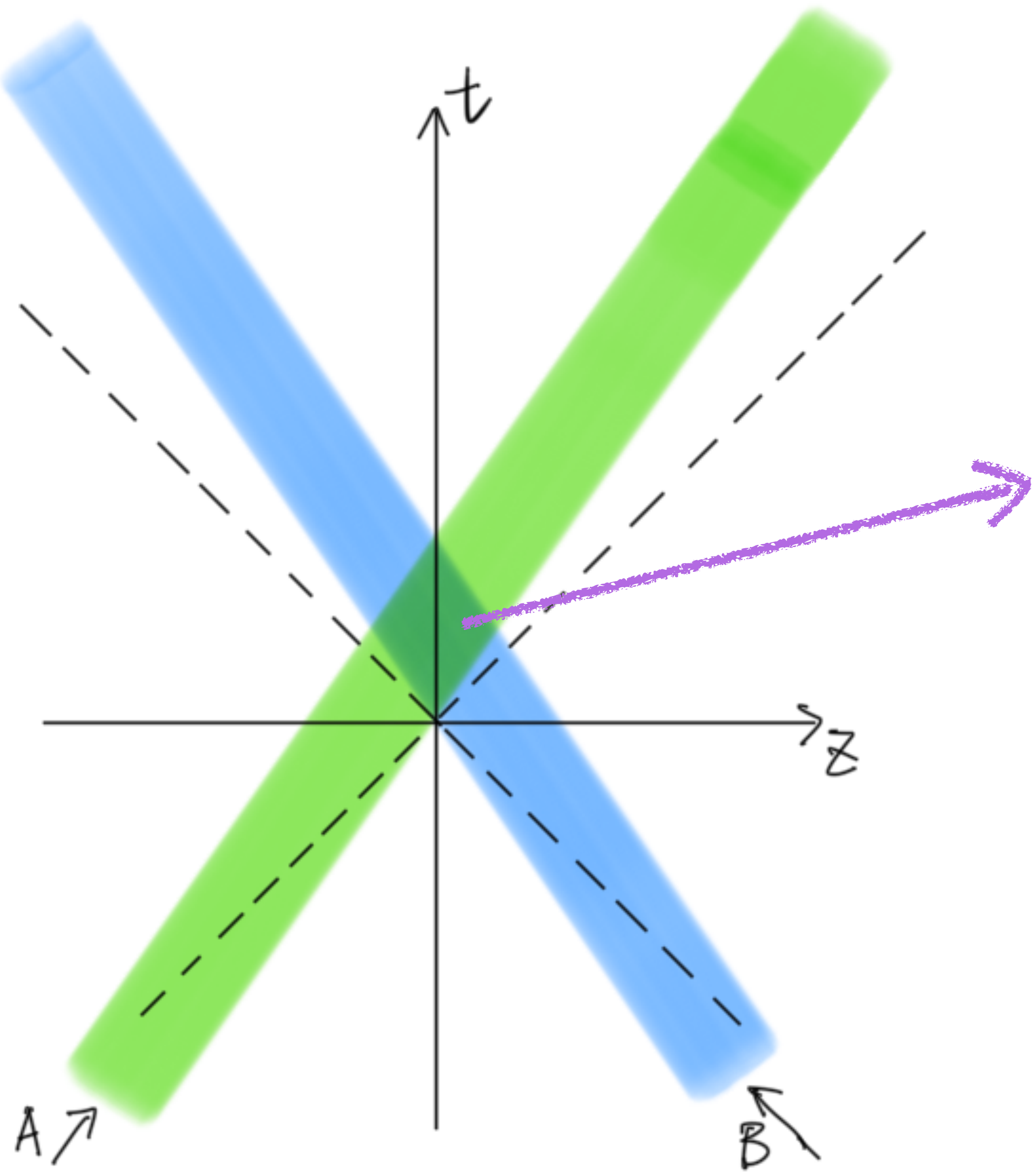
- Pre-equilibrium dynamics can play an important role

Go beyond the Bjorken approximation



- The finite widths of the colliding nuclei are taken into account

Go beyond the Bjorken approximation

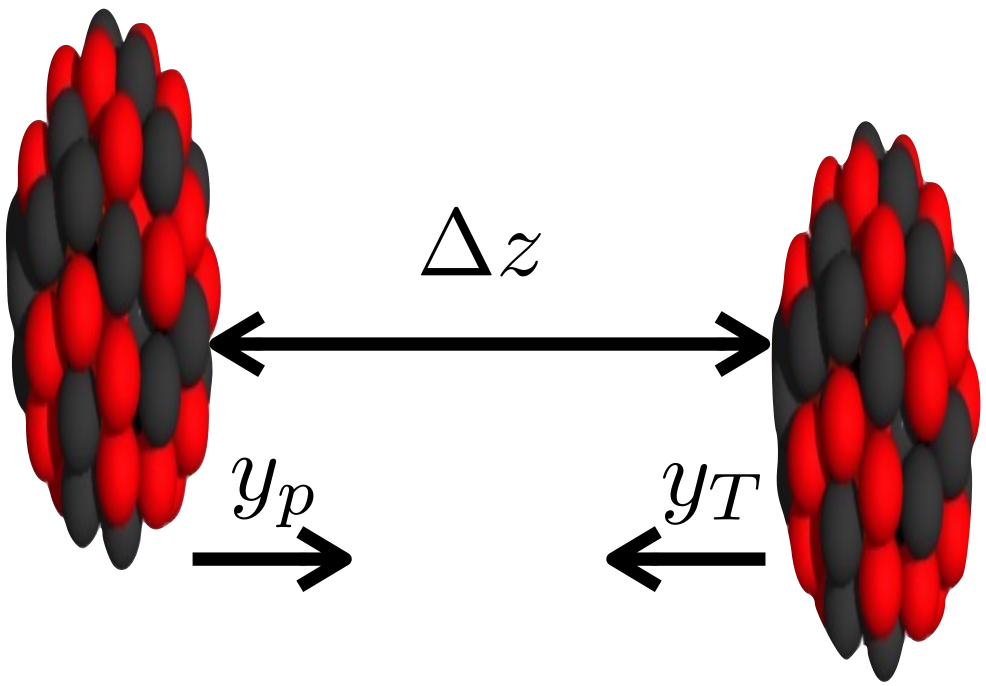


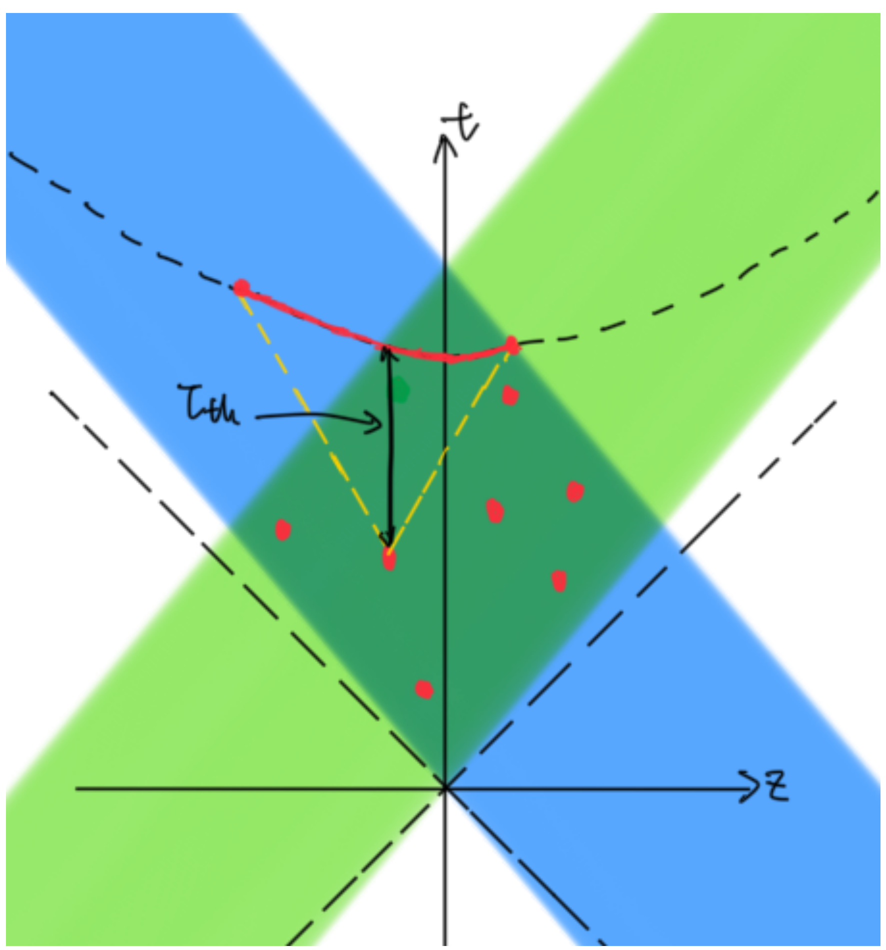
- The finite widths of the colliding nuclei are taken into account

The interaction zone is not point like

$$y \neq \eta_s$$

3D MCGlauber model with deceleration

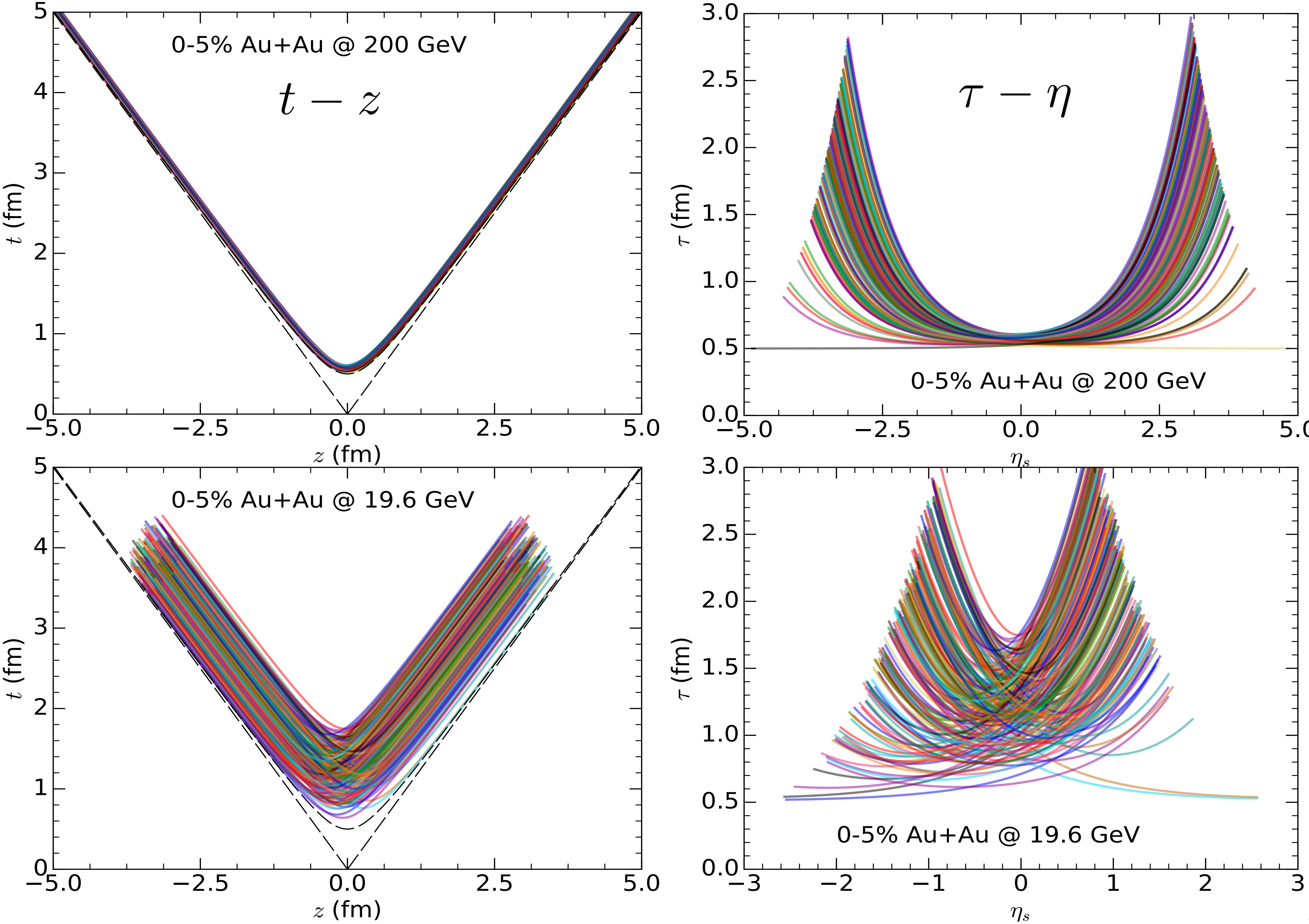




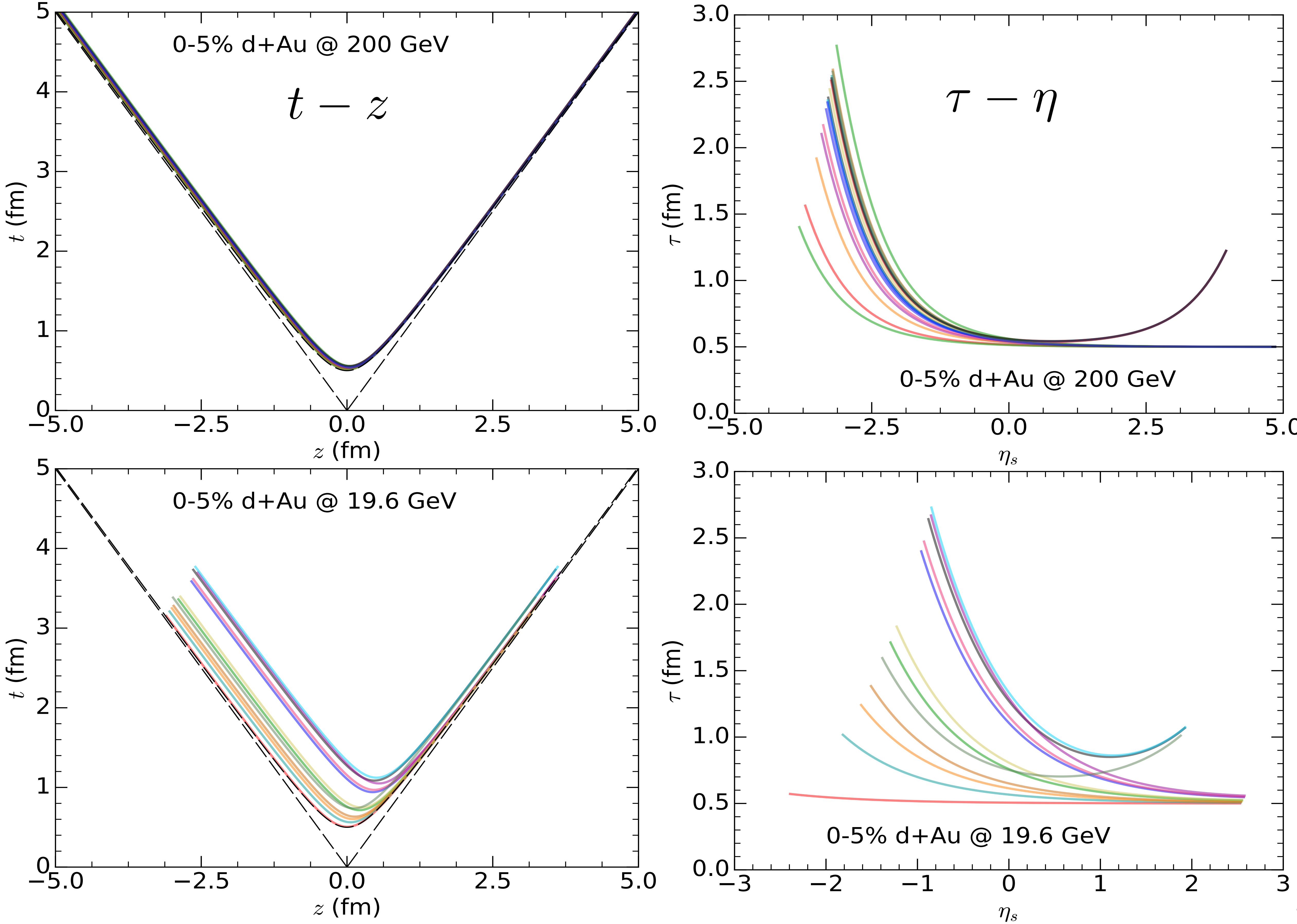
- Collision time and 3D spatial position are determined for every binary collision
- QCD strings are produced from collision points
- These strings are decelerated with a constant string tension $\sigma = 1\, GeV/fm$ for $\tau_{\mathrm{th}} = 0.5 fm$ before thermalized to medium

A. Bialas, A. Bzdak and V. Koch, arXiv:1608.07041 [hep-ph]

Strings of entropy deposition: Au+Au



Strings of entropy deposition: d+Au



Compare to the 3D MC-LEXUS model

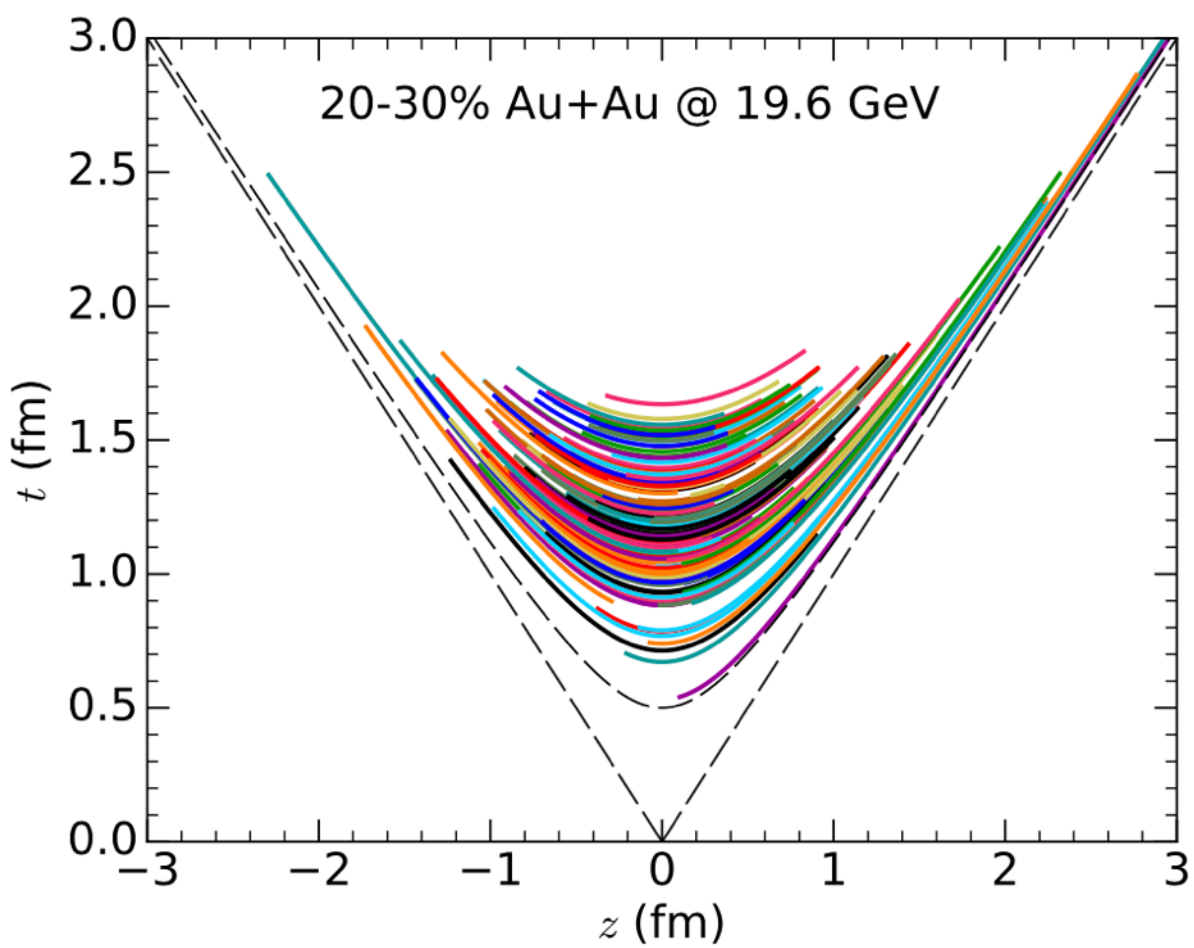
- The rapidity loss is determined by the LEXUS model

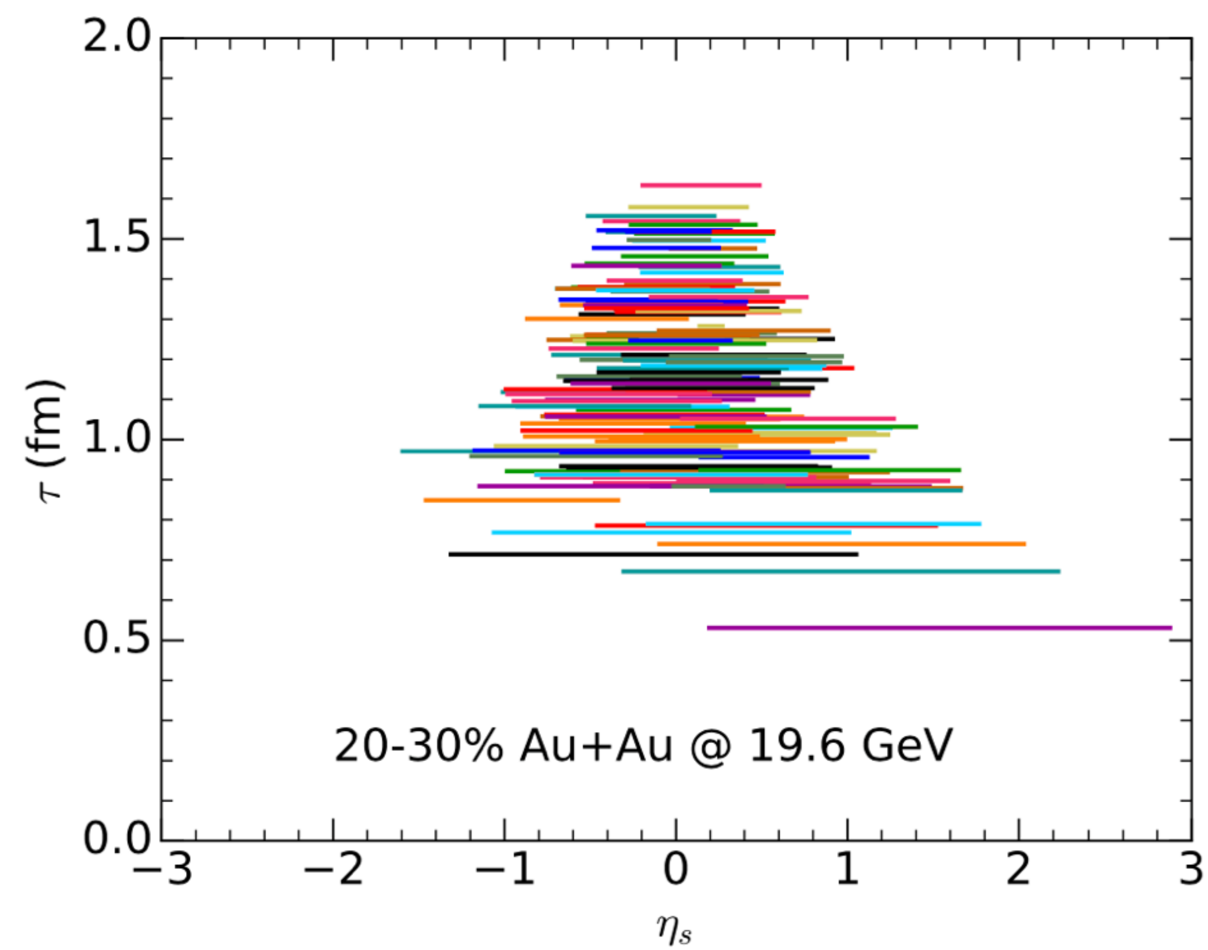
LEXUS: S. Jeon and J. Kapusta, PRC56, 468 (1997)

$$P(y_p, y_T, y) = \lambda \frac{\cosh(y - y_T)}{\sinh(y_P - y_T)} + (1-\lambda)\delta(y - y_P)$$

MC-LEXUS: A. Monnai, B. Schenke, Phys.Lett. B752 (2016) 317-321

- QCD strings are free-streaming for $\tau_{\rm th} = 0.5 fm$ before thermalizing into medium





Hydrodynamical evolution with sources

- **Before time τ_0=0.5 fm** we add up all contribution from strings and initialize the hydrodynamic simulation with them

- **After time τ_0=0.5 fm** we feed additional thermalizing strings into the hydrodynamic medium via a source term

$$\partial_\mu T^{\mu\nu} = {\color{red} J^\nu_{\mathrm{source}}}$$

$$\partial_\mu J^\mu = {\color{blue} \rho_{\mathrm{source}}}$$

where

$$J^\nu_{\mathrm{source}} = {\color{red} \delta e} u^\nu + (e+P){\color{green} \delta u^\nu}$$

$$\delta u^\nu = \frac{\Delta^\nu_\mu J^\mu_{\mathrm{source}}}{e+P}$$

heats up the system

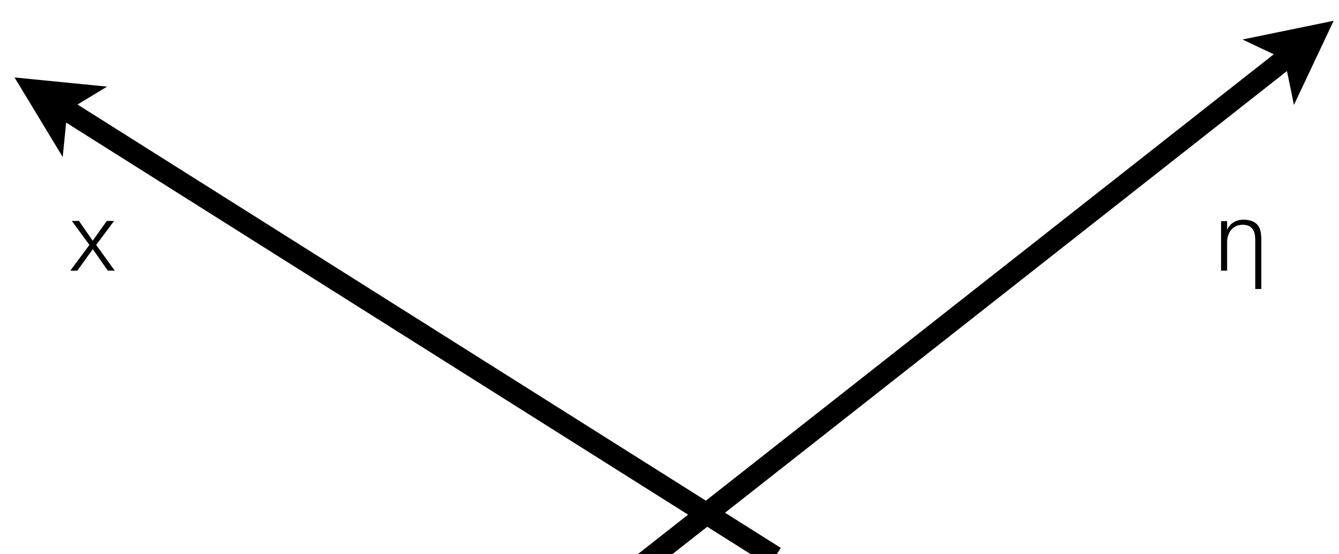
accelerates the flow velocity

${\color{blue} \rho_{\mathrm{source}}}$ feeds baryon charges into the system

Hydrodynamical evolution with sources

energy density

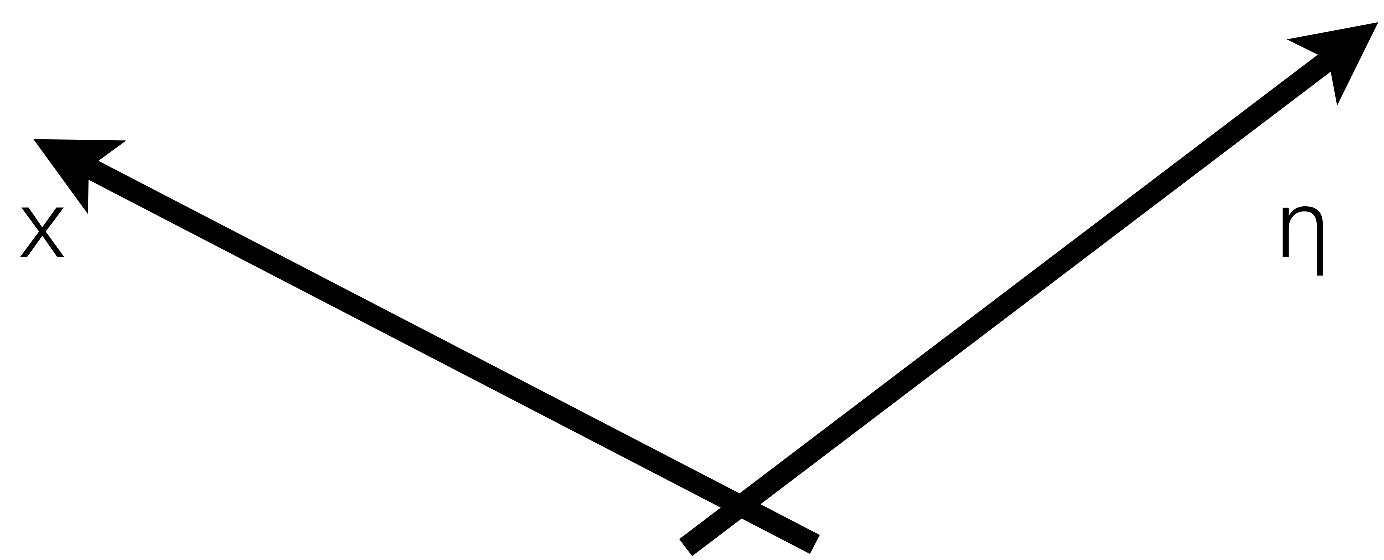
tau = 0.51 fm



Hydrodynamical evolution with sources

net baryon density

tau = 0.51 fm

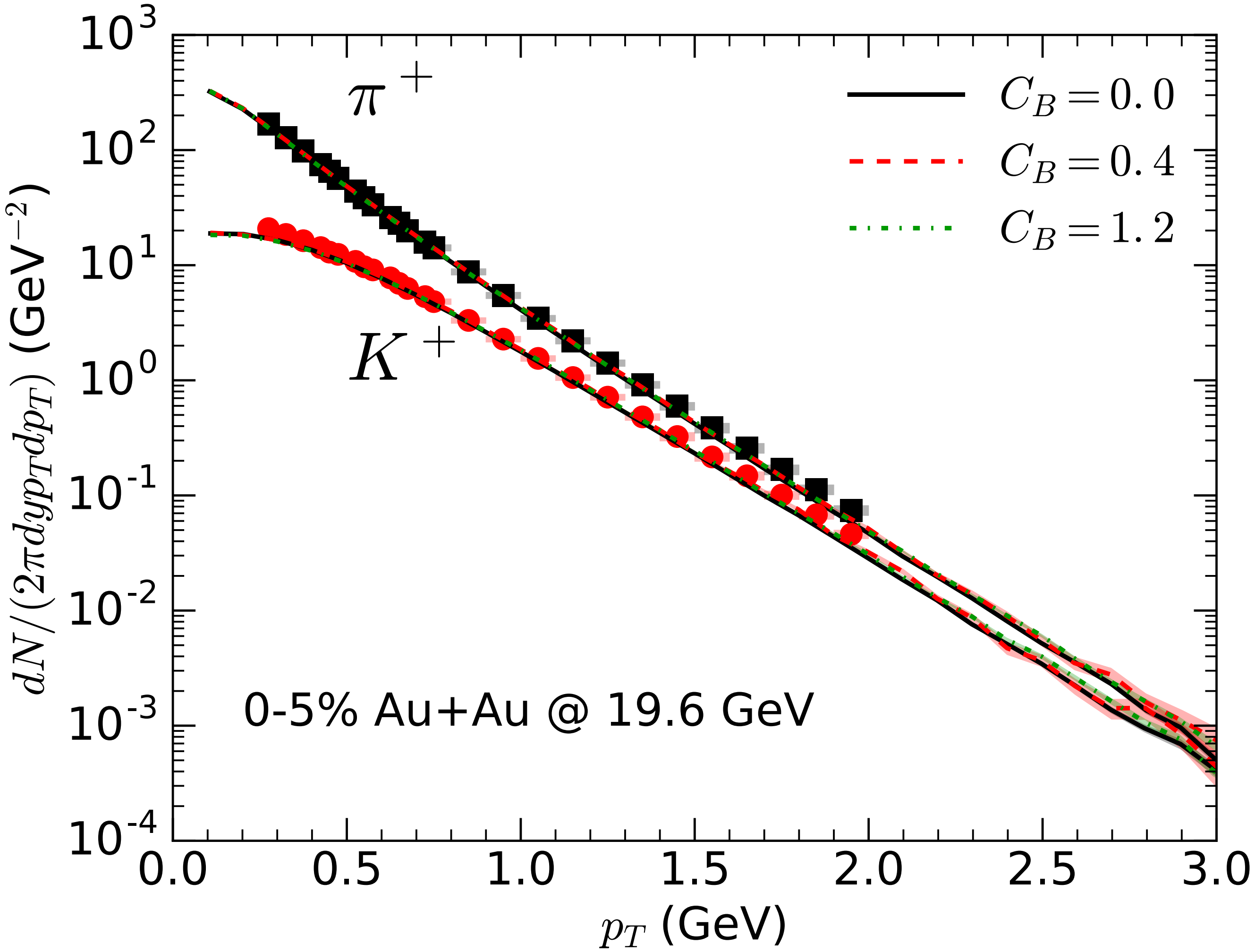


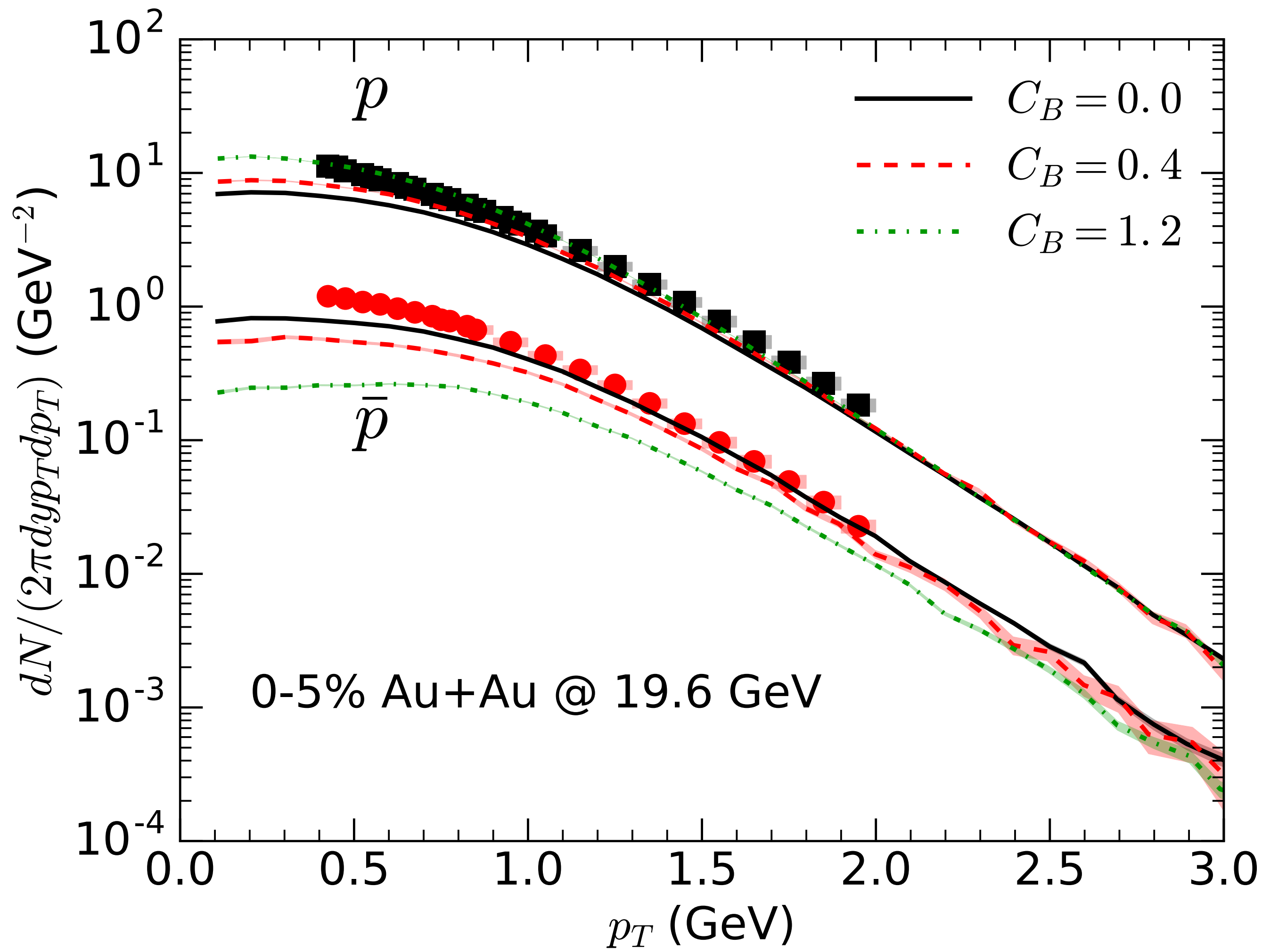
Summary and Outlook

- Hydrodynamic simulations are getting ready for the BES(II)
- Baryon diffusion has significant effects on baryon rapidity spectra
- Initial state needs dynamical component - studied various models:
  - Decelerating string model
  - MC-LEXUS model
- Hydrodynamics with source term can handle extended initialization
- Construct EoS including conditions on strangeness and isospin - can also include a critical point
- Study effect of hydrodynamic fluctuations
- Chun Shen will talk about the BES in small systems this afternoon

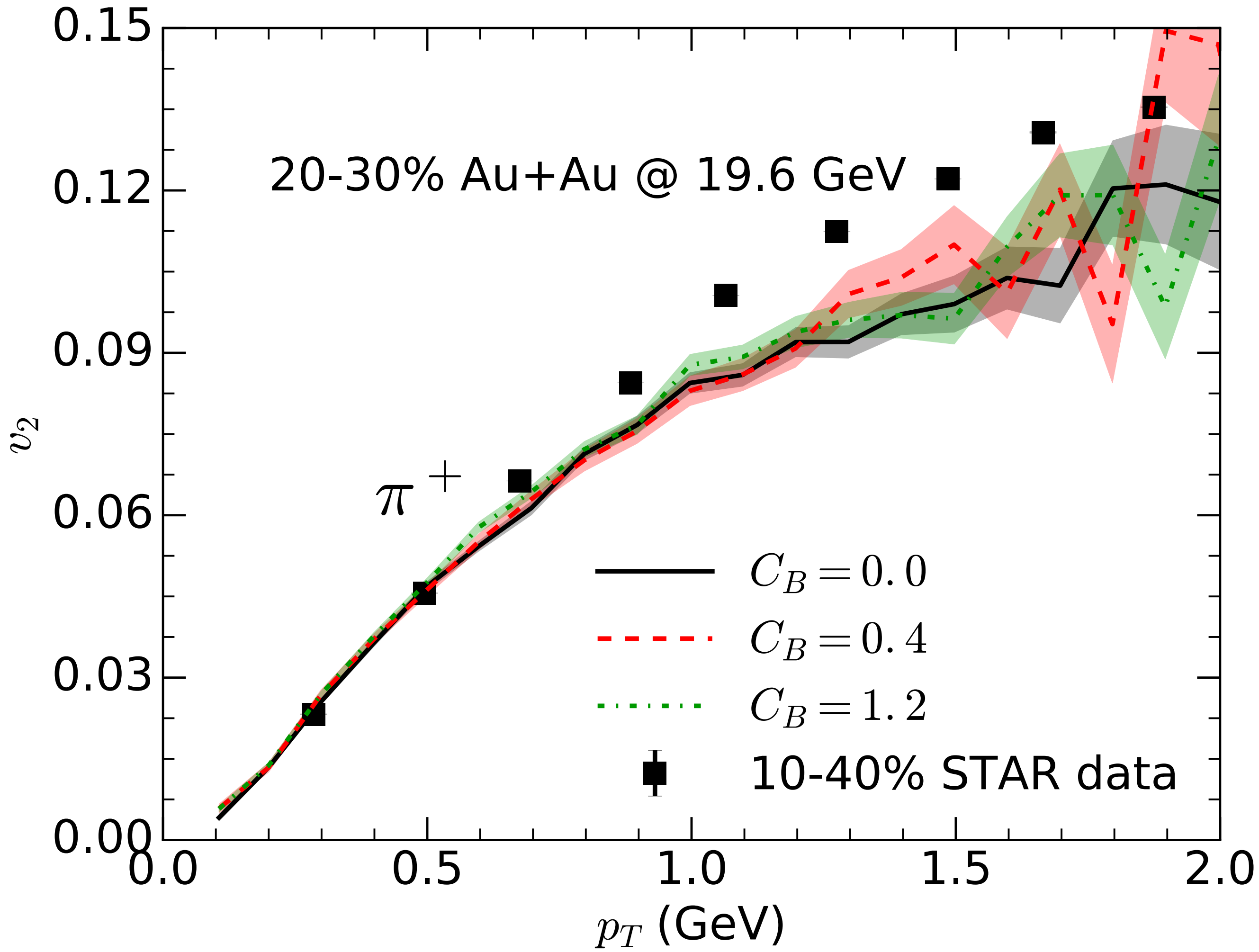
Backup

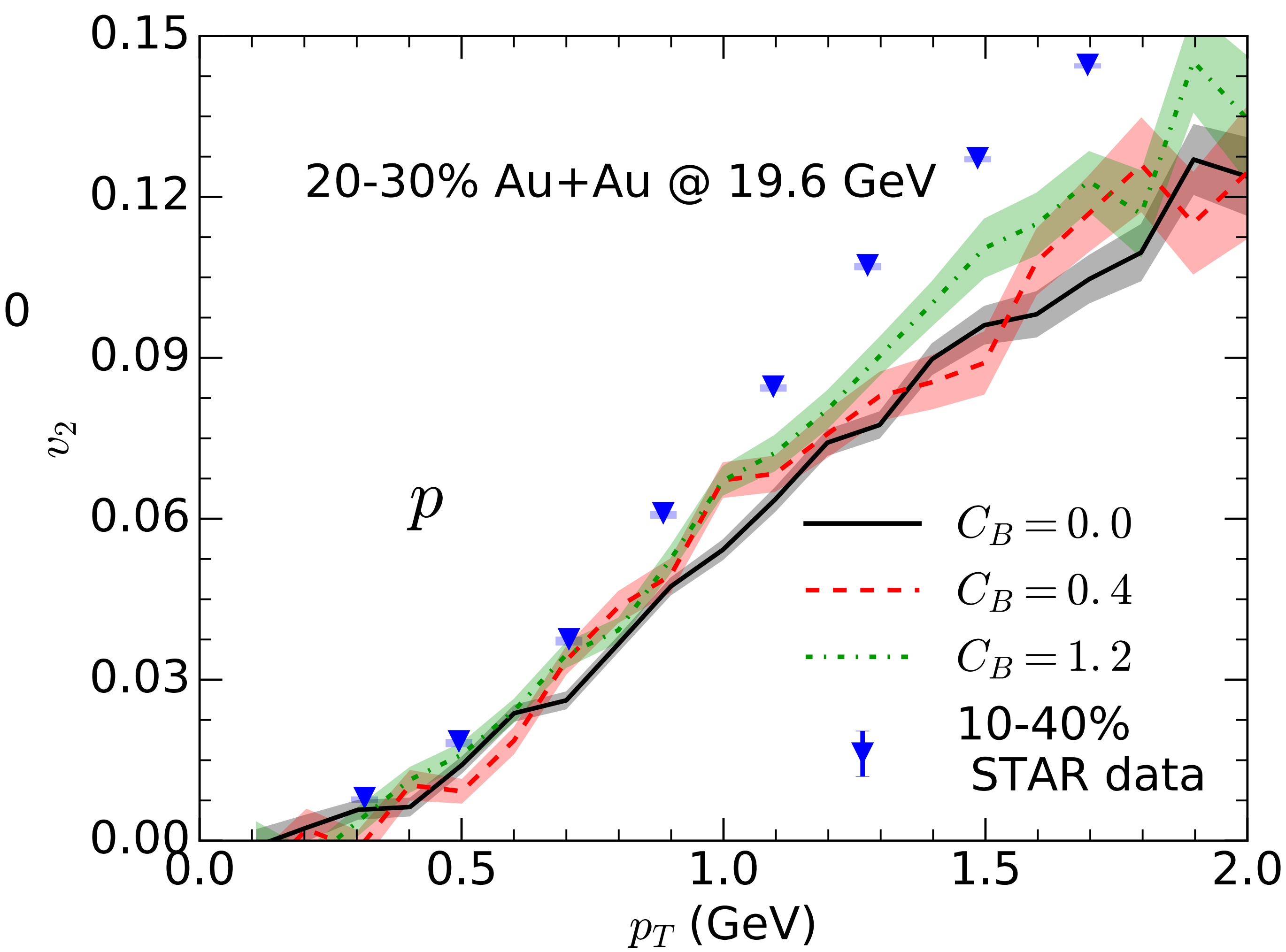
Data comparisons





Data comparisons





Constructing the equation of state (EoS)

Taylor Expansion

Cannot deal with complex Fermion determinants on lattice, so Taylor expand around zero baryon chemical potential

$$\frac{P}{T^4} = \frac{P_0}{T^4} + \frac{1}{2}\chi_B^{(2)}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{4!}\chi_B^{(4)}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^6\right]$$

because of matter-anti-mater symmetry only even powers appear similarly for energy density and entropy density

For net-baryon density we have

$$\frac{n_B}{T^3} = 0 + \chi_B^{(2)}\frac{\mu_B}{T} + \frac{1}{3!}\chi_B^{(4)}\left(\frac{\mu_B}{T}\right)^3 + \mathcal{O}\left[\left(\frac{\mu_B}{T}\right)^5\right]$$

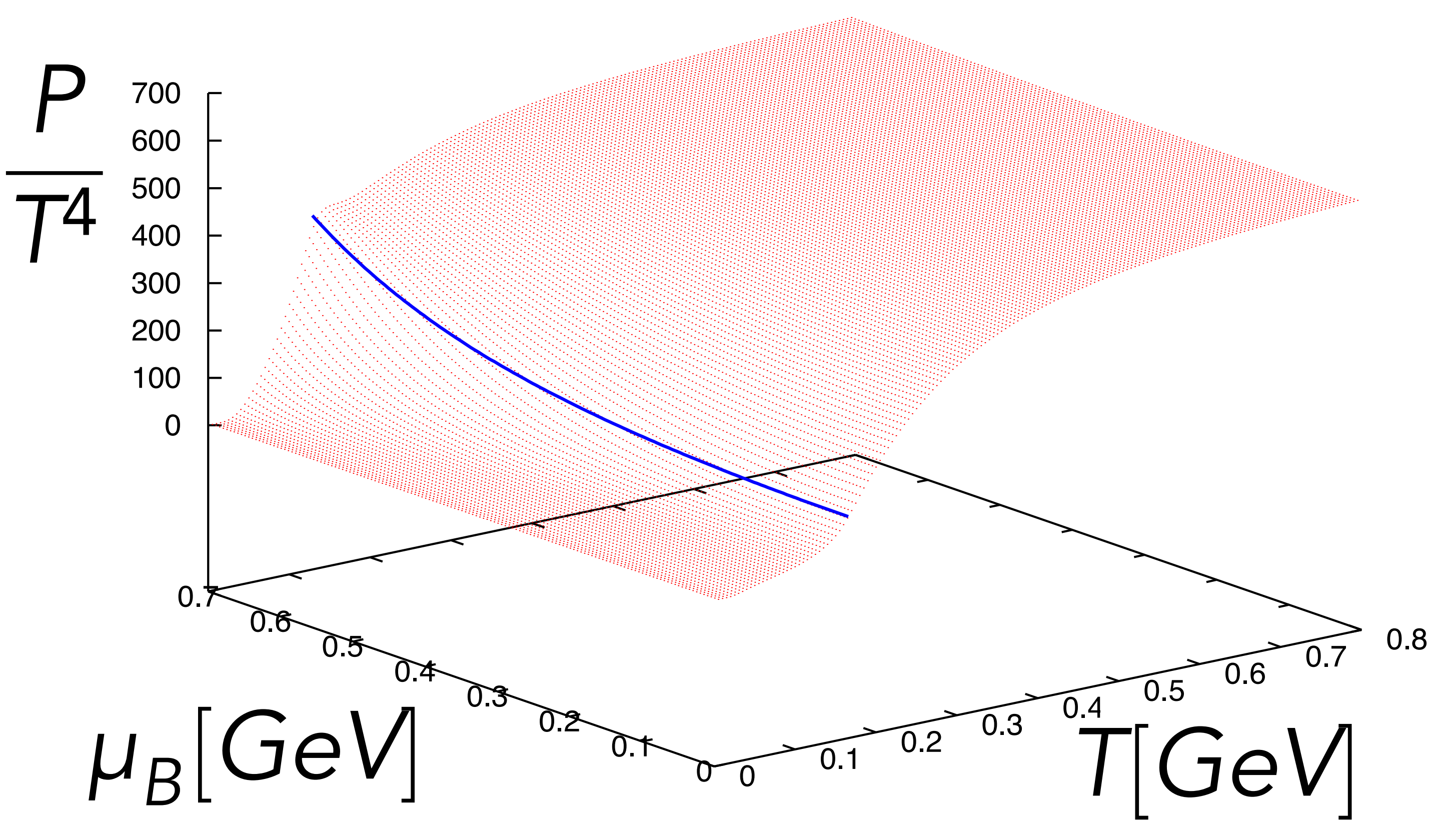
Constructing the equation of state (EoS)

Smooth matching (cross over)

We match the HRG and lattice EoS smoothly

$$\frac{P}{T^4} = \frac{1}{2}\left[1 - \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\rm HRG}(T)}{T^4} + \frac{1}{2}\left[1 + \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\rm lat}(T_s)}{T_s^4}$$

In the future one can introduce a critical point here.



T_C: connecting temperature

ΔT_C: width of overlap area

T_s: temperature shift

$$T_s = T + d[T_C(0) - T_C(\mu_B)]$$

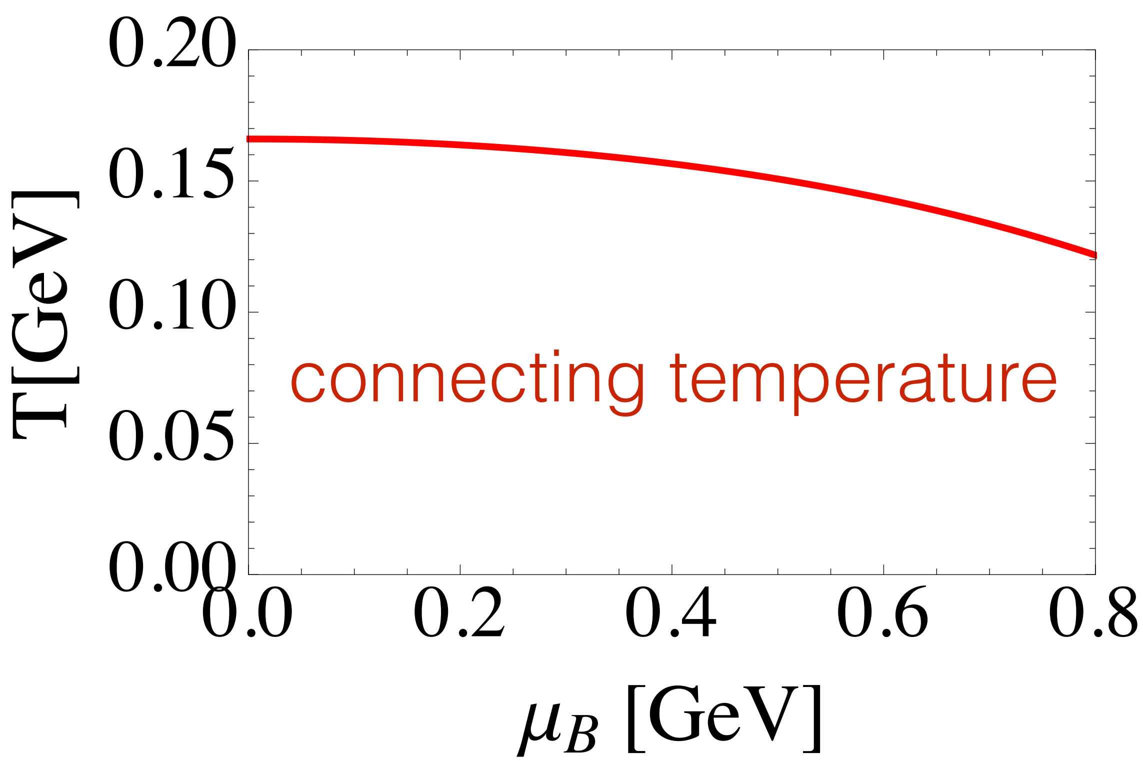
Constructing the equation of state (EoS)

Smooth matching (cross over)

We match the HRG and lattice EoS smoothly

$$\frac{P}{T^4} = \frac{1}{2}\left[1 - \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\mathrm{HRG}}(T)}{T^4} + \frac{1}{2}\left[1 + \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\mathrm{lat}}(T_s)}{T_s^4}$$

$$T_C(\mu_B) = 0.166\mathrm{GeV} - c(0.139\mu_B^2 + 0.053\mu_B^4)$$



based on the chemical freeze-out line (c=1)

Cleymans et al, PRC73, 034905 (2006)

For the connecting line we use c=d=0.4, ΔT_C=0.1 $T_C(0)$

Constructing the equation of state (EoS)

Smooth matching (cross over)

We match the HRG and lattice EoS smoothly

$$\frac{P}{T^4} = \frac{1}{2}\left[1 - \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\rm HRG}(T)}{T^4} + \frac{1}{2}\left[1 + \tanh\frac{T - T_C(\mu_B)}{\Delta T_C}\right]\frac{P_{\rm lat}(T_s)}{T_s^4}$$

Parameters P_0^{lat} and $\chi_B^{(2)}$ are determined from the lattice:

Borsanyi et al, JHEP1011, 077 (2010)
Borsanyi et al, JHEP1201, 138 (2012)

$\chi_B^{(4)}$ is obtained from the ratio $\chi_B^{(4)}/\chi_B^{(2)}$ in a HRG and parton gas model

δf corrections in the presence of net baryons

A. Monnai, T. Hirano, PRC80, 054906 (2009); Nucl. Phys. A847, 283 (2010)

Grad's 14 moment method

$$\delta f^i = -f_0^i(1 \pm f_0^i)(b_i \underbrace{\varepsilon^B_\mu}_{4} p_i^\mu + \underbrace{\varepsilon_{\mu\nu}}_{10} p_i^\mu p_i^\nu)$$

particle i's baryon quantum number

ε^B_μ and $\varepsilon_{\mu\nu}$ are determined by the self-consistency conditions

$$\delta T^{\mu\nu} = \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i = -\Pi \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\delta N_B^\mu = \sum_i \int \frac{b_i g_i d^3p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i = \overset{=0}{\cancel{V_B^\mu}}$$ (no baryon diffusion)

δf corrections in the presence of net baryons

A. Monnai, T. Hirano, PRC80, 054906 (2009); Nucl. Phys. A847, 283 (2010)

Grad's 14 moment method

$$\delta f^i = -f_0^i(1 \pm f_0^i)(b_i \varepsilon_\mu^B p_i^\mu + \varepsilon_{\mu\nu} p_i^\mu p_i^\nu)$$

After tensor decomposition and one finds

$$\varepsilon_\mu^B = D_\Pi \Pi u_\mu$$

$$\varepsilon_{\mu\nu} = (B_\Pi \Delta_{\mu\nu} + \tilde{B}_\Pi u_\mu u_\nu)\Pi + B_\pi \pi_{\mu\nu}$$

where the coefficients are computed in kinetic theory

We parametrize them as functions of T and μ_B

Note: Results of net baryon density are very sensitive to accuracy of the bulk-δf parametrization

Lexus model S. Jeon and J. Kapusta, PRC56, 468 (1997)

Input: The distribution of outgoing nucleons in a high energy nucleon-nucleon collision is flat in longitudinal momentum or a hyperbolic cosine (symmetric about the CM) in rapidity

M. A. Abolins, G. A. Smith, Z. Ming Ma, Eugene Gellert, and A. B. Wicklund
Phys. Rev. Lett. 25, 126

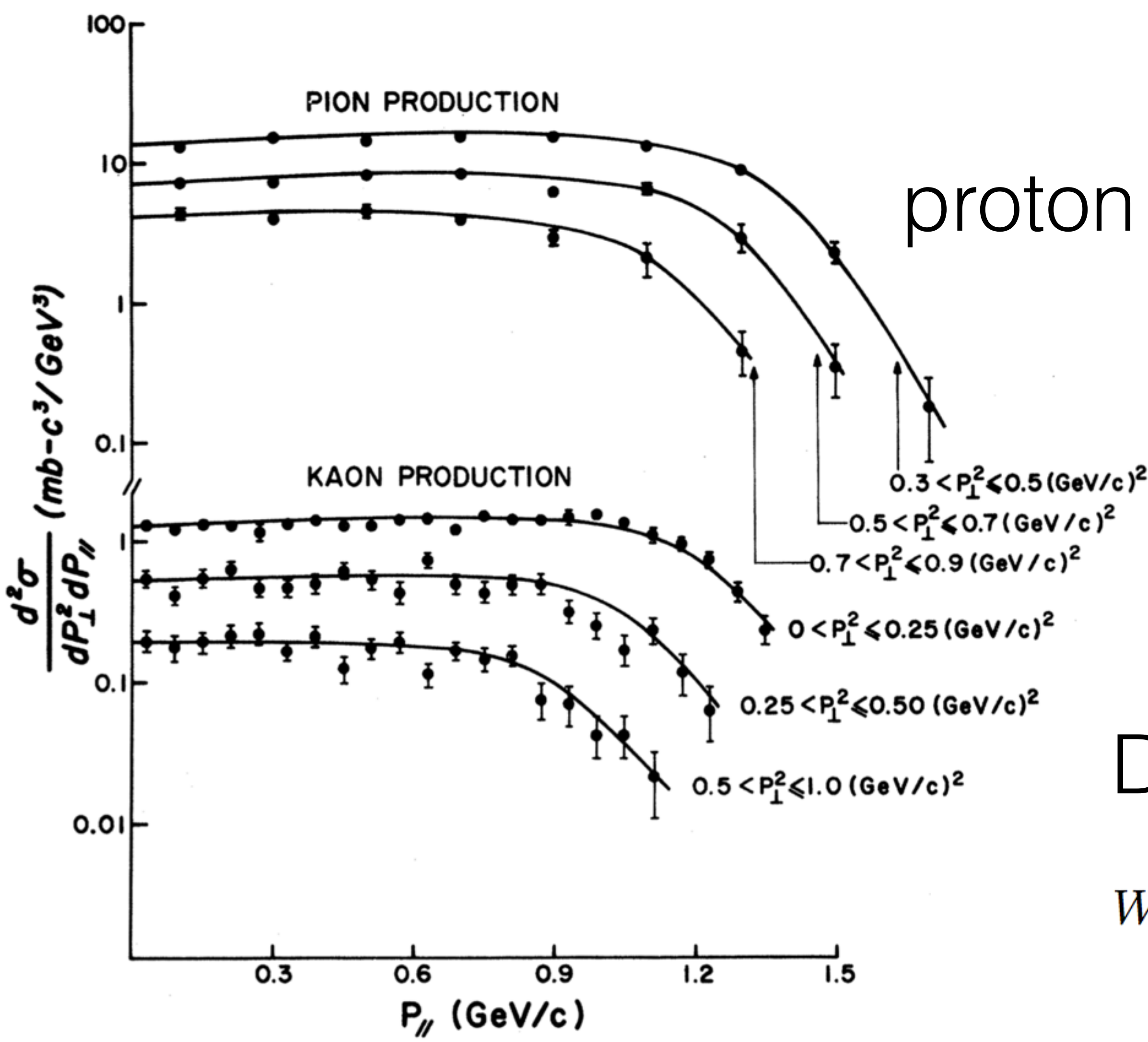


FIG. 1. Proton c.m. longitudinal-momentum distributions for various values of $P_\perp^2$. The solid lines are free-hand fits to the data.

proton long. momentum distributions

Distribution of outgoing projectile

$$W_{11}^P(y) = Q(y, y_0, y_0 - y) = \lambda \frac{\cosh y}{\sinh y_0} + (1-\lambda)\delta(y_0 - y)$$

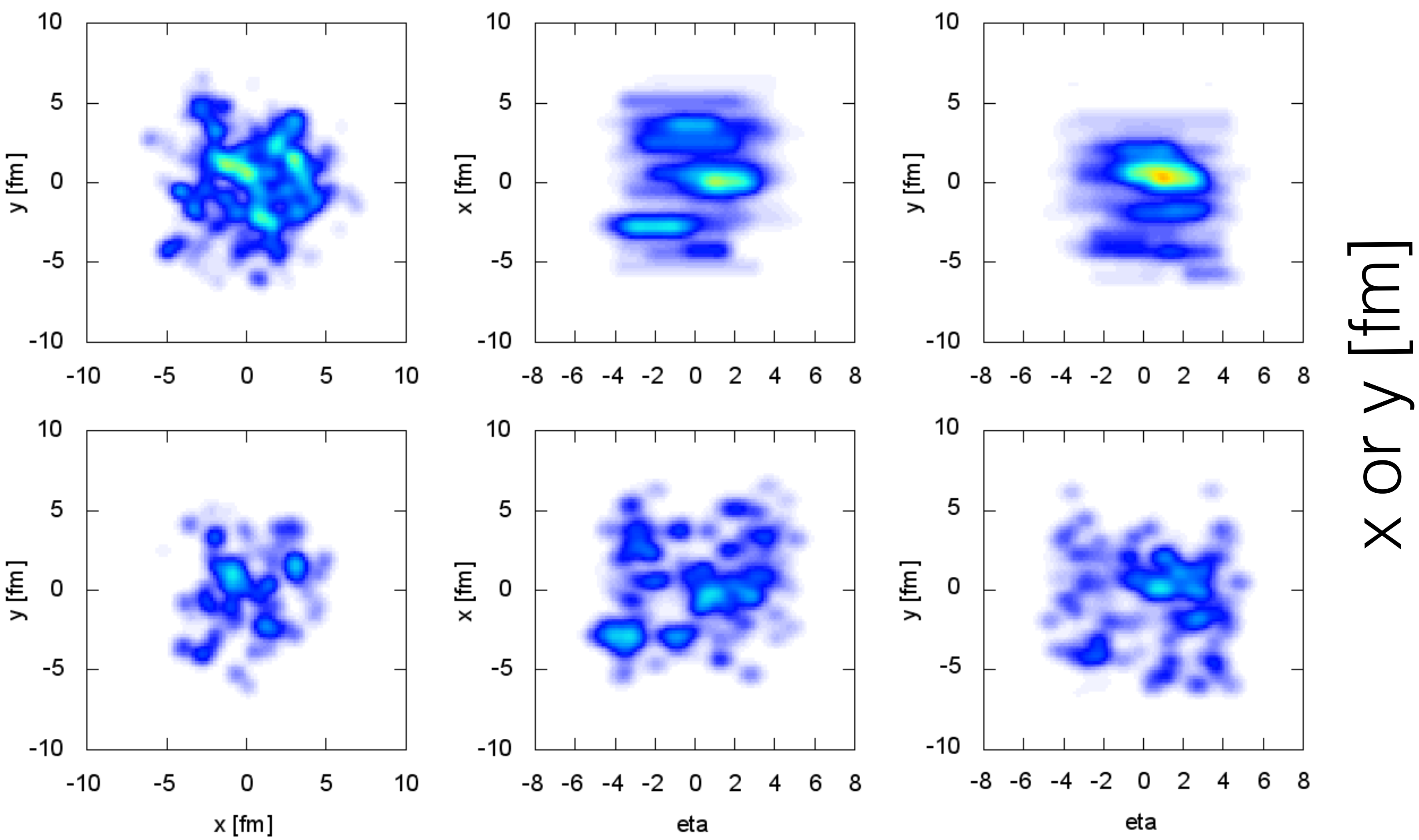
Event-by-event baryon- and entropy density

Deposit entropy density (fluctuating with NBD) between the wounded quarks using Gaussian profile in the transverse plane and a constant distribution (with Gaussian edges) in rapidity

$\sqrt{s} = 200\mathrm{GeV}$

energy density

baryon density



x[fm]

rapidity

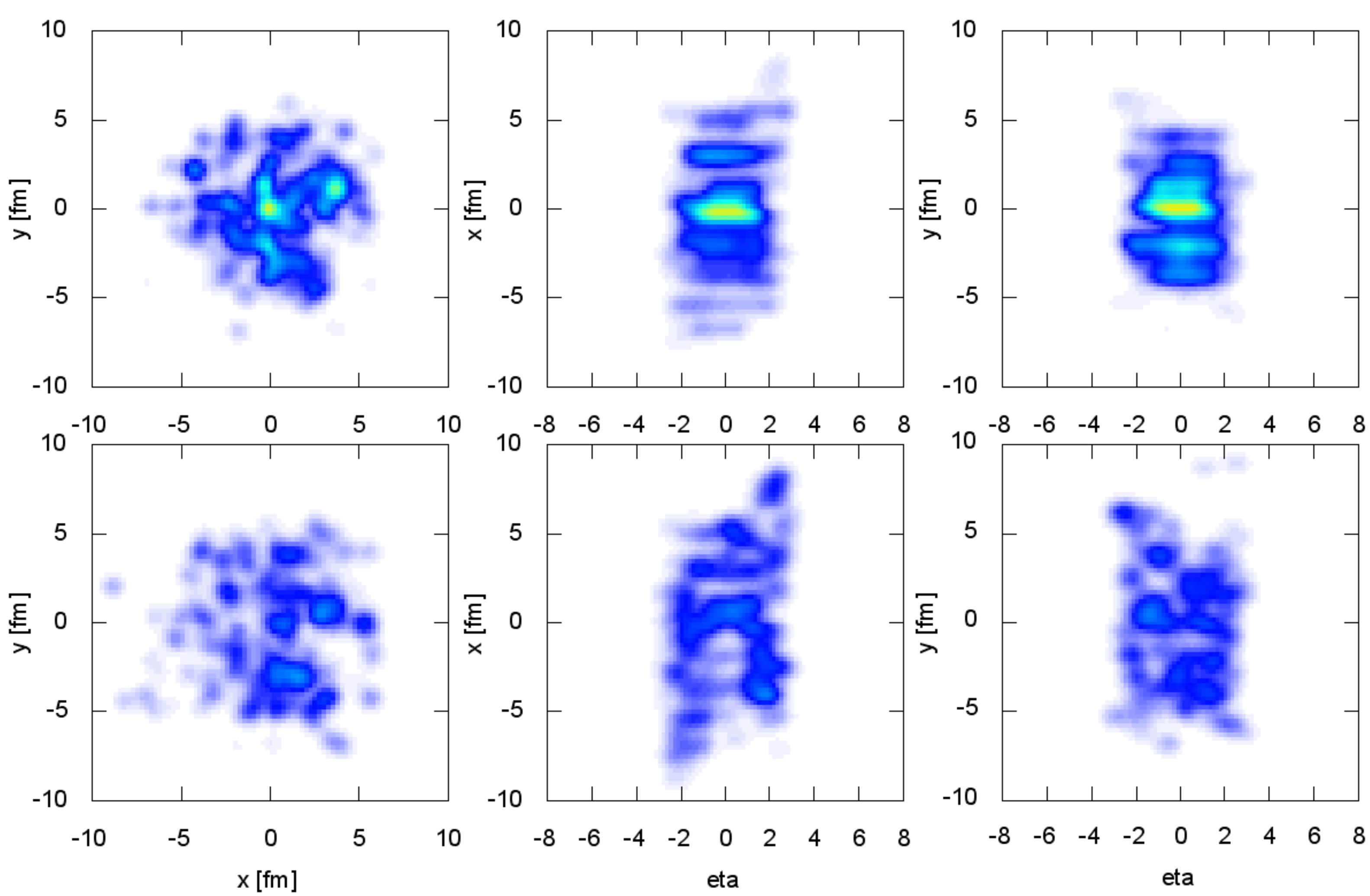
Event-by-event baryon- and entropy density

Deposit entropy density (fluctuating with NBD) between the wounded quarks using Gaussian profile in the transverse plane and a constant distribution (with Gaussian edges) in rapidity

$\sqrt{s} = 19.6\mathrm{GeV}$

energy density

baryon density

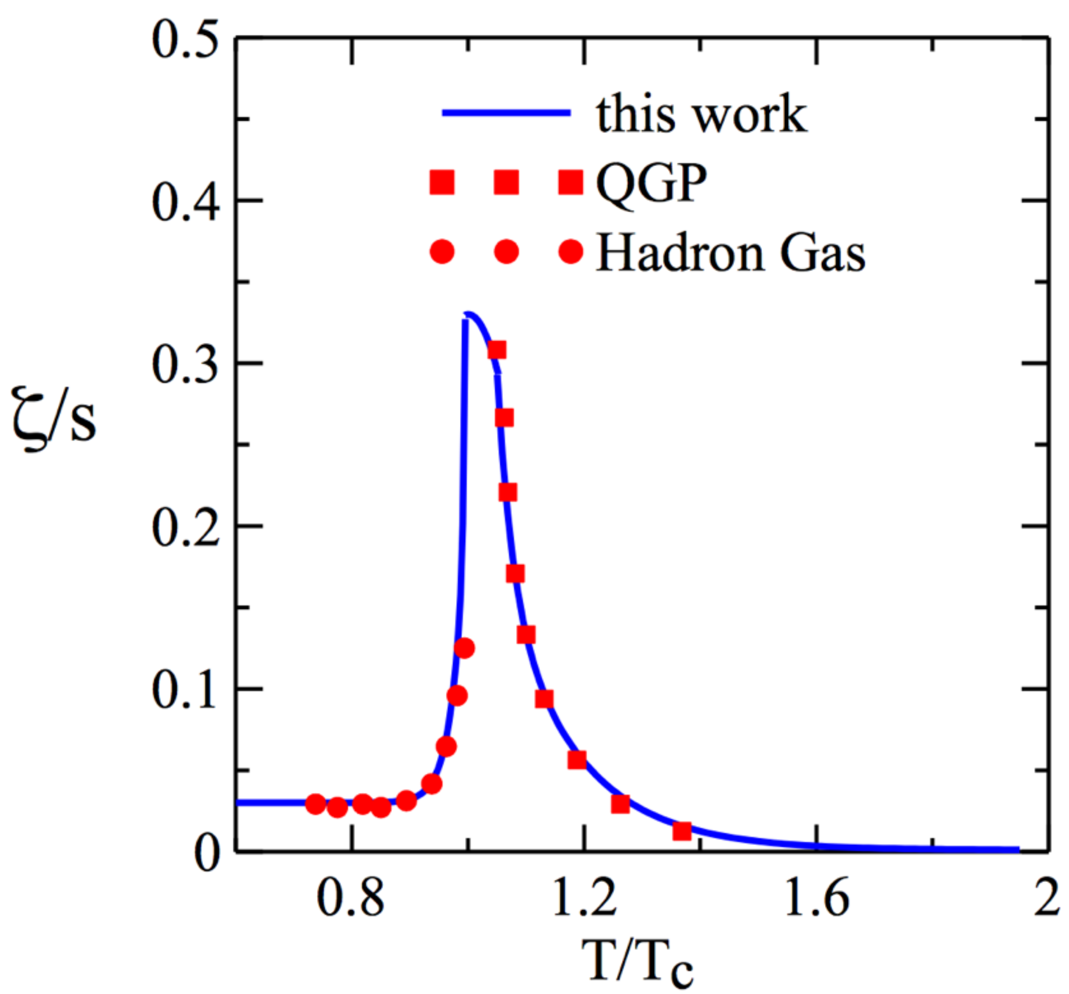


x[fm]   rapidity   x or y [fm]

Viscosities

In the calculations presented here we use:

- shear viscosity (constant or with *T* dependence to be defined)
- bulk viscosity:



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